ENVIRONMENTAL GEOTECHNICS

NATURAL AND ARTIFICIAL SLOPES and GENERAL SLOPE STABILITY CONCEPTS

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Definition of LANDSLIDE

The downward falling or sliding of a mass of soil, detritus, or rock on or from a steep slope.

Introduction to SLOPE STABILITY

Slope stability problems have been faced throughout history when men and women or nature has disrupted the delicate balance of natural soil slopes.

Need to understand analytical methods, investigative tools, and stabilization methods to solve slope stability problems.

AIMS OF SLOPE STABILITY ANALYSIS

The primary purpose of slope stability analysis is to contribute to the safe and economic design of excavations, embankments, earth dams, landfills, and spoil heaps.

The aims of slope stability analyses are:

(1) To understand the **development and form** of natural slopes and the processes responsible for different natural features.

(2) To assess the stability of slopes under **short-term** (often during construction) and **long-term** conditions.

AIMS OF SLOPE STABILITY ANALYSIS

(3) To assess the **possibility of landslides** involving natural or existing engineered slopes.

(4) To analyze landslides and to understand **failure mechanisms** and the influence of environmental factors.

(5) To enable the redesign of failed slopes and the planning and design of **preventive and remedial measures**, where necessary.

(6) To study the effect of **seismic loadings** on slopes and embankments.

NATURAL AND ARTIFICIALS SLOPES

The analysis of slopes takes into account a variety of factors relating to topography, geology, and material properties, often relating to whether the slope was naturally formed or engineered.

Natural slopes that have been stable for many years may suddenly fail because of changes in topography, seismicity, groundwater flows, loss of strength, stress changes, and weathering.

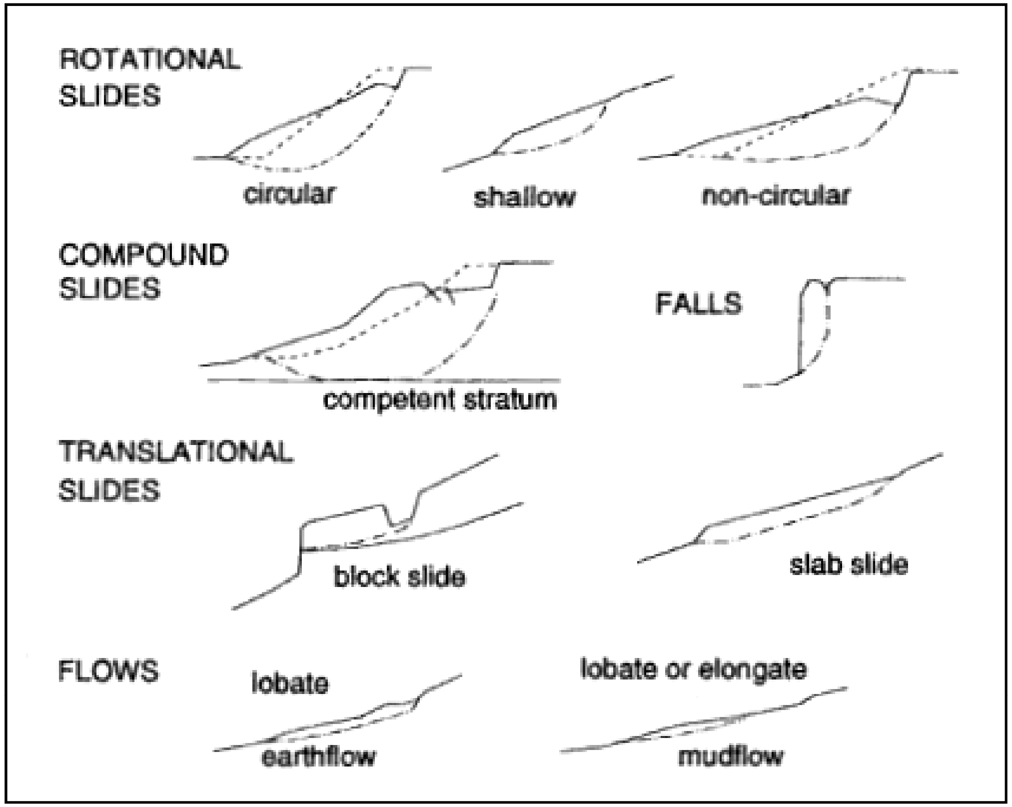
Significant uncertainty exists about the stability of a natural slope.

Knowing that old slip surfaces exist in a natural slope makes it easier to understand and predict the slope's behavior. The shearing strength along these slip surfaces is often very low because prior movement has caused slide resistance to peak and gradually reduce to residual values.

Engineered slopes may be considered in three main categories: embankments, cut slopes, and retaining walls. As these slopes are man-made less uncertainty exists about their stability.

MODES OF FAILURE

Slope failures are usually due either to a sudden or gradual **loss of strength** by the soil or to a **change in geometric conditions**, for example, steepening of an existing slope.



Typical slides that can be expected to occur in soil slopes:

(1) Falls (free fall, topple)

(2) Slides

2a. Planar or translational

2b. Rotational or slump

2c. Block or wedge slides

2d. Lateral spreading

2e. Debris slide

(3) Flows

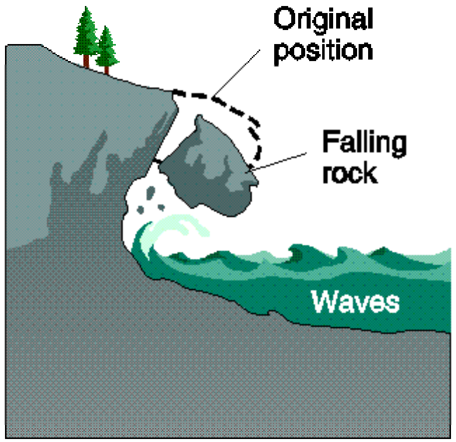
(4) Creep

(5) Compound slides (combination of previous types)

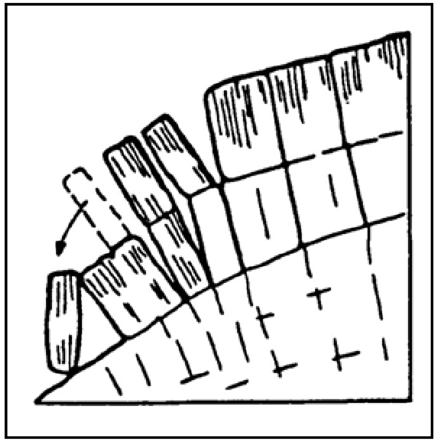
MODES OF FAILURE

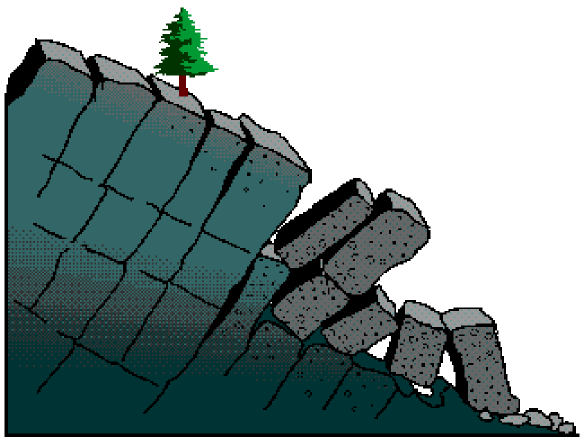
A CLASSIFICATION OF SLOPE FAILURES		
Type	**Form**	**Definition**
Falls	Free fall	Sudden dislodgement of single or multiple blocks of soil or rock which fall in free descent.
	Topple	Overturning of a rock block about a pivot point located below its center of gravity.
Slides	Rotational or slump	Relatively slow movement of an essentially coherent block (or blocks) of soils, rock, or soil-rock mixtures along some well-defined arc-shaped failure surface.
	Planar or translational	Slow to rapid movement of an essentially coherent block (or blocks) of soil or rock along some well-defined planar failure surface.
	Block slide	A single block moving along a planar surface.
	Wedge slide	Block or blocks moving along intersecting planar surfaces.
	Lateral spreading	A number of intact blocks moving as separate units with differing displacements.
	Debris slide	Soil-rock mixtures moving along a planar rock surface.
Flows	Debris Sand Silt Mud Soil	Soil or rock-soil debris moving as a viscous fluid or slurry. Usually terminating at distances far beyond the failure zone; resulting from excessive pore pressures.
Creep		Slow, imperceptible downslope movement of soil or soil-rock mixtures.
Complex		Involves combinations of the above, usually occurring as a change from one form to another during failure with one form predominant.

Falls



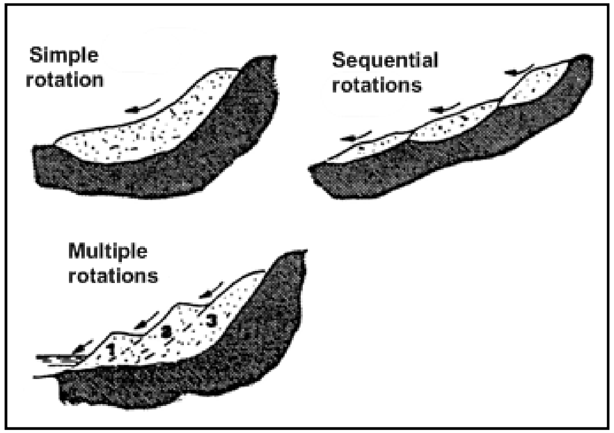
Free fall

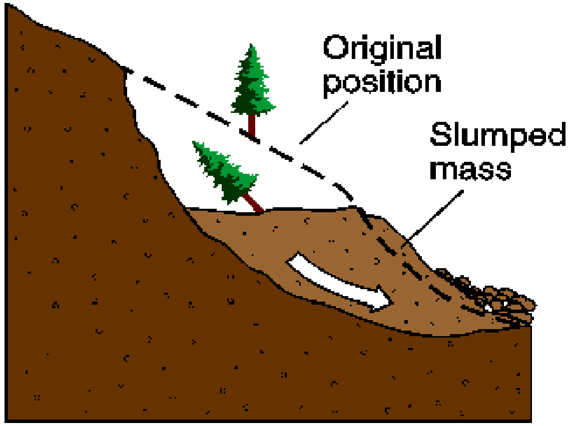




Topple

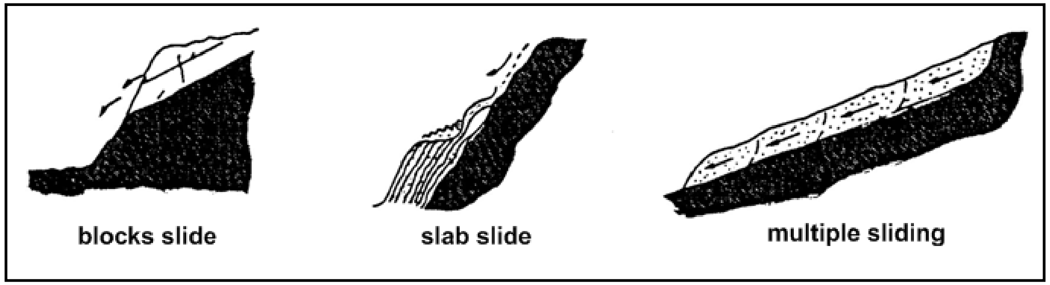
Slides

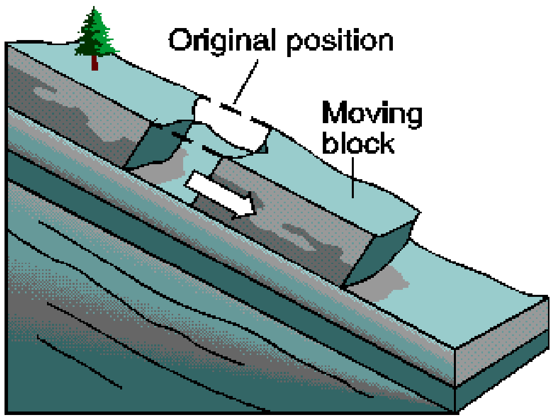


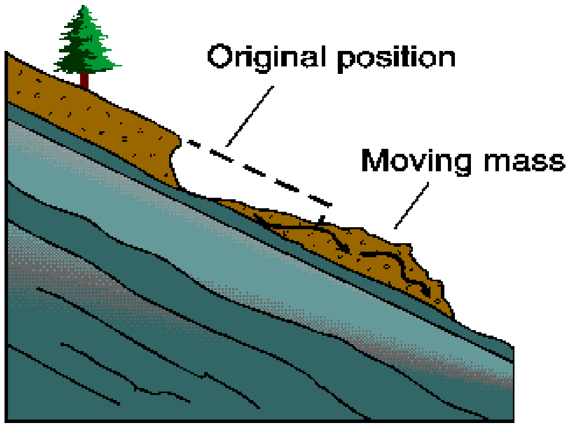


Rotational Slide

Usually occurs in slopes consisting of homogeneous materials



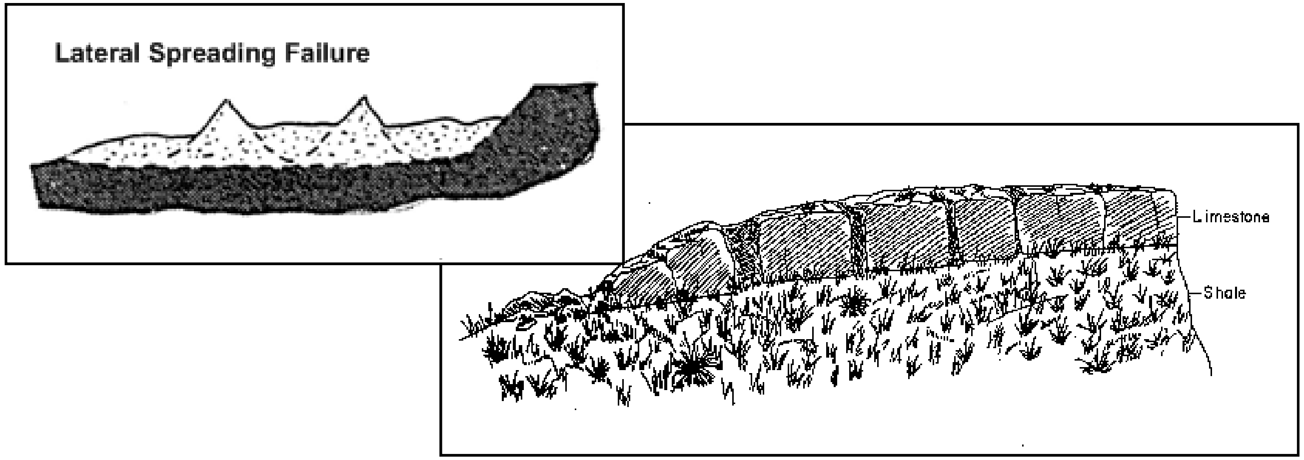




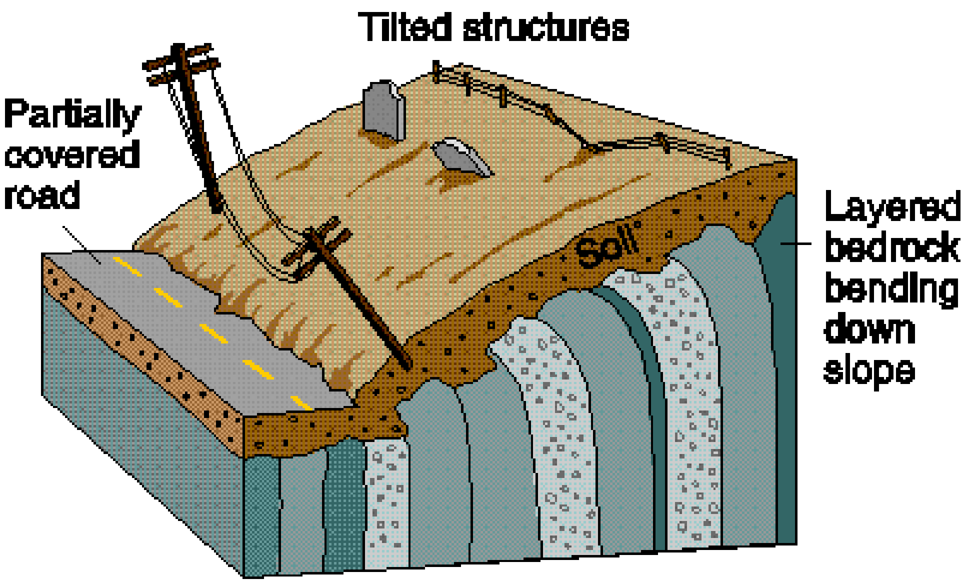
Translational Slide

Usually occurs in shallow soils overlying relatively stronger materials

Lateral spreading & Debris flow



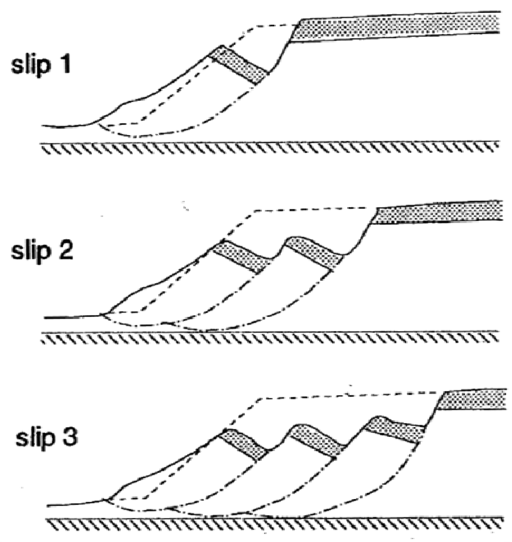
Lateral spreading



Debris flow

As most soils are generally heterogeneous, noncircular surfaces, consisting of a combination of planar and curved sections, are most likely.

Often, retrogressive failures consisting of multiple curved surfaces can occur in layered soils. Such failures are typical where the first slip tends to oversteepen the slope, which then leads to additional failures.



Such failures are typical where the first slip tends to oversteepen the slope, which then leads to additional failures.

Geologic Factors Controlling Shape of Potential Failure Surface

Geologic Conditions	Potential Failure Surface
Cohesionless soils Residual or colluvial soils over shallow rock Stiff fissured clays and marine shales within the upper, highly weathered zone	Translational with small depth/length ratio
Sliding block Interbedded dipping rock or soil Faulted or slickensided material Intact stiff to hard cohesive soil on steep slopes	Single planar surface
Sliding blocks in rocky masses Weathered interbedded sedimentary rocks Clay shales and stiff fissured clays Stratified soils Sidehill fills over colluvium	Multiple planar surfaces
Thick residual and colluvial soil layers Soft marine clays and shales Soft to firm cohesive soils	Circular or cylindrical shape

CASE HISTORIES (1)

Toppling failure

Planar Sliding

CASE HISTORIES (2)

Lateral spreading

Tully Valley Landslide,

Phoenix, New York, 1993

CASE HISTORIES (3)

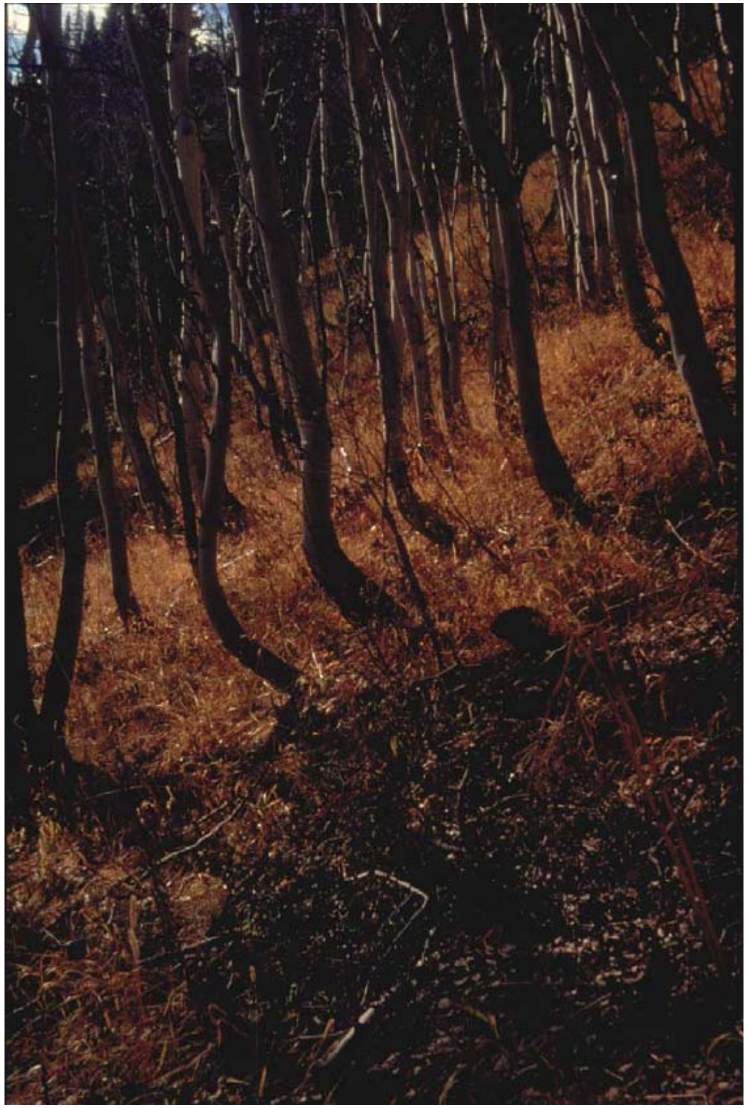
Cones of dejection and screes at the feet of Canadian Rockies

CASE HISTORIES (4)

S. Salvador, El Salvador, 2001

Debris Flow

CASE HISTORIES (5)



Creep

The roots of trees are embedded in the stable soil, while the trunks follow the downward slow movement of the superficial cover.

CASE HISTORIES (6)

Rotational landslide

with debris-flow

La Conchita, California, 1995

CASE HISTORIES (7)

Fort St. John, Alberta, Canada, 2001

Roto-traslational landslide

Young River Landslide, Canada

FACTORS INFLUENCING SLOPE STABILITY

The main items required to evaluate the stability of a slope are:

(1) Shear strength of the soils

(2) Slope geometry

(3) Pore pressures or seepage forces

(4) Loading and environmental conditions

(1) The shear strengths should be provided as undrained strength, s_u, or the more typical Mohr-Coulomb parameters, c and ϕ.

(2) The slope geometry may be known for existing, natural slopes or may be a design parameter for embankments and cut slopes.

(3) A major contributor to many slope failures is the change in effective stress caused by pore water pressures. These tend to alter the shear strength of the soil along the shear zone

FACTOR OF SAFETY (FOS) CONCEPTS

Function of the FOS: to account for uncertainty, and thus to guard against ignorance about the reliability of the items that enter into the analysis, such as, strength parameters, pore pressure distribution, and stratigraphy.

The lower the quality of the site investigation, the higher the desired FOS should be.

The FOS used in design will vary with material type and performance requirements.

The required FOSs (nonseismic) are usually in the 1.25 to 1.5 range. Higher factors may be required if there is a high risk of loss of life or uncertainty regarding the pertinent design parameters. Likewise, lower FOSs may be used if the engineer is confident of the accuracy of input data and if the construction is being monitored closely.

FACTOR OF SAFETY– First Definition

In most limit equilibrium analyses, the shear strength required along a potential failure surface to just maintain stability is calculated and then compared to the magnitude of available shear strength. In this case the FOS is assumed to be constant for the entire failure surface.

This average FOS will be given by the ratio of available to required shear strength:

$$\tau_{req} = \frac{s_u}{F}$$

for total stresses

$$\tau_{req} = \frac{c'}{F_c} + \frac{\sigma' \cdot \tan\phi}{F_\phi}$$

for effective stresses

FACTOR OF SAFETY– First Definition

The adoption of F_c and F_ϕ allows different proportions of the cohesive (*c'*) and frictional (ϕ') components of strength to be mobilized along the failure surface.

However, most limit equilibrium methods assume F_c and F_ϕ , implying that the same proportion of the *c'* and ϕ' components are mobilized at the same time along the shear failure surface.

FACTOR OF SAFETY– Second Definition

Another definition of FOS often considered is the ratio of total resisting forces to total disturbing (or driving) forces for planar failure surfaces or the ratio of total resisting to disturbing moments, as in the case for circular slip surfaces.

Realize that these different values of the FOS obtained using the three methods, that is, mobilized strength, ratio of forces, or ratio of moments, will not give identical values for $c - \phi$ soils.

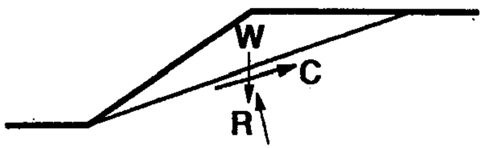
FACTOR OF SAFETY

LIMIT EQUILIBRIUM

$$FOS = \frac{s_u}{\tau_{required}} \qquad \text{(Total Stress)}$$

$$FOS = \frac{c' + \sigma' \tan\phi'}{\tau_{required}} \qquad \text{(Effective Stress)}$$

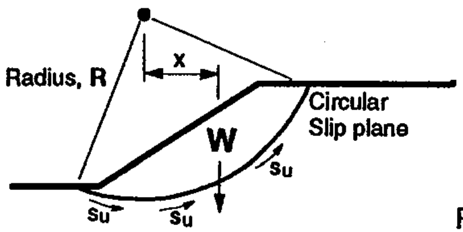
First definition



FORCES

$$FOS = \frac{\text{Summation of resisting force}}{\text{Summation of mobilized force}}$$

Second definition



MOMENTS

$$FOS = \frac{\text{Resisting moment}}{\text{Overturning moment}} = \frac{R \int s_u \, ds}{W\,x}$$

PORE WATER PRESSURES

If an effective stress analysis is to be performed, pore water pressures will have to be estimated at relevant locations in the slope. These pore pressures are usually estimated from groundwater conditions that may be specified by one of the following methods:

(1) Phreatic Surface: This surface, or line in two dimensions, is defined by the free groundwater level. Delineated, in the field, by using open standpipes as monitoring wells.

(2) Piezometric Data: Specification of pore pressures at discrete points, within the slope, and use of an interpolation scheme to estimate the required pore water pressures at any location. Determined from field piezometers, a manually prepared flow net or a numerical solution using finite differences or finite elements.

PORE WATER PRESSURES - r_u

(3) Pore Water Pressure Ratio: This is a popular and simple method for normalizing pore water pressures measured in a slope according to the definition:

$$r_u = \frac{u}{\sigma_v}$$

where *u* is the pore pressure and σ_v, is the total vertical subsurface soil stress at depth *z*. Effectively, the r_u, value is the ratio between the pore pressure and the total vertical stress at the same depth.

This factor is easily implemented, but the major difficulty is associated with the assignment of the parameter to different parts of the slope.

It is usually reserved for estimating the FOS value from slope stability charts or for assessing the stability of a single surface.

PORE WATER PRESSURES

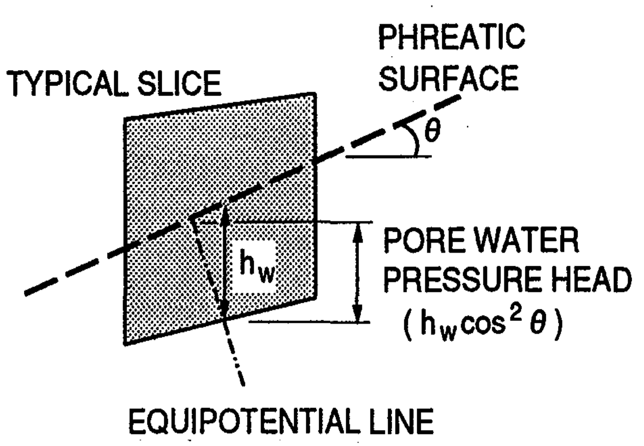
(4) Piezometric Surface: This surface is defined for the analysis of a unique, single failure surface. This approach is often used for the back analysis of failed slopes.

Note that a piezometric surface is not the same as a phreatic surface, as the calculated pore water pressures will be different for the two cases.

(5) Constant Pore Water Pressure: This approach may be used if the engineer wishes to specify a constant pore water pressure in any particular soil layer.

PORE WATER PRESSURES – Phreatic surface

If a phreatic surface is defined, the pore water pressures are calculated for the steady-state seepage conditions.



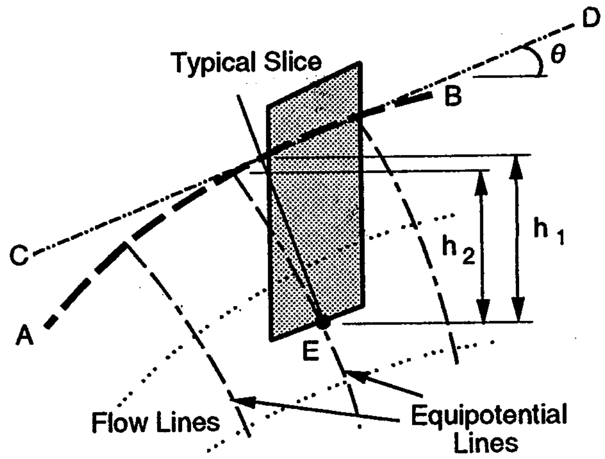
This concept is based on the assumption that all equipotential lines are straight and perpendicular to the segment of the phreatic surface passing through a slice-element in the slope.

PORE WATER PRESSURES – Phreatic surface

Thus if the inclination of the phreatic surface segment is θ, and the vertical distance between the base of the slice and the phreatic surface is *h*, the pore pressure is given by:

$$u = \gamma_w \cdot \left(h_w \cdot \cos^2 \theta\right)$$

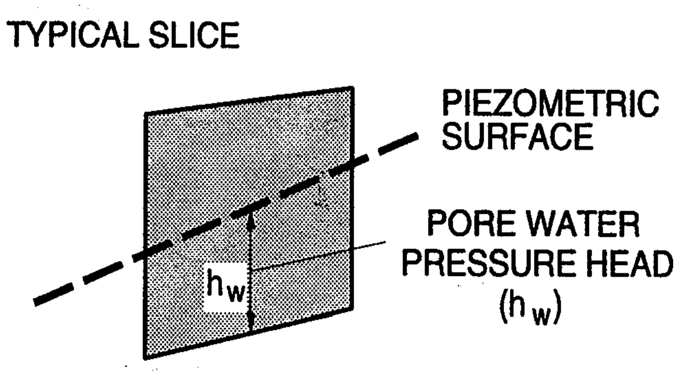
This is a reasonable assumption for a sloping straight-line phreatic surface, but will provide higher or lower estimates of pore water pressure for a curved (convex) phreatic surface.



AB - Actual Phreatic Surface
CD - Assumed Inclination of Phreatic Surface within Slice

PORE WATER PRESSURES – Piezometric surface

Note that the vertical distance (elevational head) is taken to represent the pressure.



There are some computer programs that oversimplify the analysis by misinterpreting a phreatic surface as a piezometric surface. With this erroneous assumption, the overestimated pore pressure head is incorrectly taken as the vertical distance between the phreatic surface and base of slice.

Negative Pore Pressures

There may be cases where an engineer wishes to use negative pore pressures to take advantage of the apparent cohesive strength available due to suction within the soil in the slope.

The influence of suction should be included by increasing the total cohesion according to the measured values of matric suction within the slope.

In some cases, actual negative pore pressures have been used in slope analysis to increase the shear strength of the soil.

This method is not recommended, as it only affects the frictional component via the $(\sigma - u) \tan \phi$ term and may not generate reliable values of strength.

LIMIT EQUILIBRIUM METHOD – L.E.M.

There are numerous methods currently available for performing slope stability analyses. The majority of these may be categorized as limit equilibrium methods.

All the procedures currently used are based on the L.E.M. and have the following assumptions in common:

- Coulomb's failure criterion is satisfied along the assumed failure surface, which may be a straight line, circular arc, logarithmic spiral, or other irregular surface.
- Plain strain conditions are supposed.
- The actual strength of the soil is compared with the value required for the equilibrium of the soil mass and this ratio is a measure of the safety factor.

LIMIT EQUILIBRIUM METHOD – L.E.M.

In this method soils are treated as rigid-plastic materials and due to this assumption the analysis does not consider deformations. Therefore, this method allows to only condition at the onset of failure.

These methods are very similar to the kinematic approach, but frequently the restrictions of a kinematically admissible mechanisms are ignored.

Limit Equilibrium Methods can be subdivided in two principal categories:

- Methods that consider only the whole free body (Culmann method, friction circle method).
- Methods that divide the free body into many vertical slices and consider equilibrium of each slide (method of slices).

LIMIT EQUILIBRIUM METHOD – L.E.M.

Failure surface	Method	Assumptions	Author
Planar	Infinite slope	Constant slope angle, infinite extension	Taylor (1948)
	Culmann	Planar failure surface intercepting the foot of the slope	Culmann (1966)
Intersected Planes	Wedge or wedges	Single block or sliding blocks	Hoek-Bray (1981) NAVFAC (1982)
	Wedges Method	Lateral forces acting on sides of each slice are equal	Fellenius (1936)
Circular Arc	Friction Circle	Direction of the resultant of the normal and frictional component of strength mobilized along the failure surface is tangent to a concentric circle of radius $R_f = R \sin \phi_m$	Taylor (1948)
	Bishop	System of forces acting on lateral interslice surfaces	Bishop (1955)
	Modified Bishop	Simplified system of forces acting on lateral interslice surfaces	Bishop (1955)
	Spencer	Interslice forces are considered parallel	Spencer (1967, 1981)
Irregular	Morgenstern-Price	Use of general equilibrium equations	Morgenstern-Price (1965)
	Janbu	Consider interslice forces	Janbu (1954)

INFINITE SLOPE ANALYSIS

The most simple way to analyze slope stability.

It is used when:

- A slope extends for a relatively long distance and has a consistent subsoil profile.

The failure plane is parallel to the surface of the slope and the limit equilibrium method can be applied readily.

Infinite slopes can be studied under several configurations:

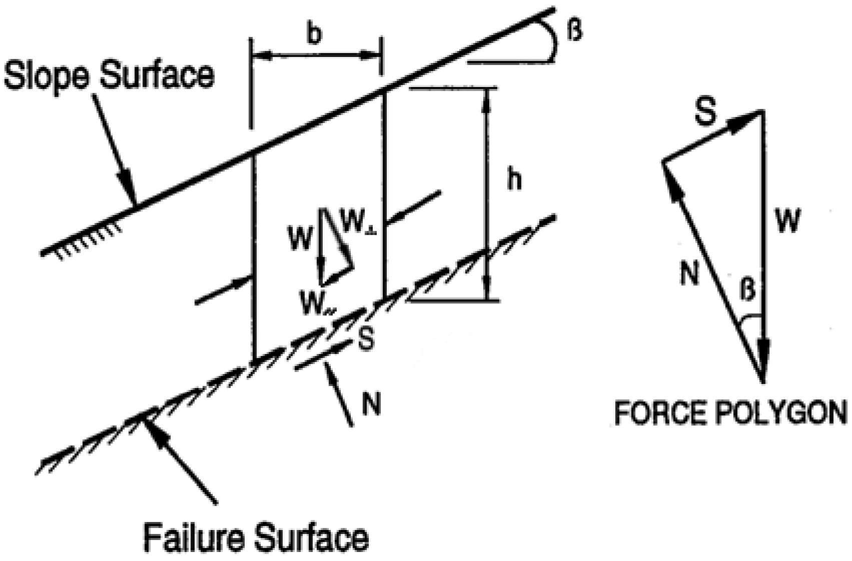
- Infinite slope in dry sand.
- Infinite slope in $c - \phi$ soil with seepage: seepage parallel to the slope.
- Infinite slope in $c - \phi$ soil with seepage: horizontal seepage (i.e. rapid dewatering of canals).

INFINITE SLOPE ANALYSIS – Dry Soil

Dry Soil

Interslice tensions on the lateral sides of each element are equal, so the soil mass moves like a continuum.

The normal ($W_{\perp}$) and driving ($W_{//}$) forces are determined:



$W_{\perp} = W \cos\beta = \gamma\ b\ h\ 1 \cos\beta$

$W_{//} = W \sin\beta = \gamma\ b\ h\ 1 \sin\beta$ where β is the constant inclination of the slope

INFINITE SLOPE ANALYSIS – Dry Soil

The available frictional strength along the failure plane will depend on ϕ and is given by

$$S = c'\, b \sec\beta + N \tan\phi = c'\, b \sec\beta + W_{\perp} \tan\phi$$

If we consider the FOS as the ratio of available strength to strength required to maintain stability (limit equilibrium), the FOS will be given by

$$FOS = \frac{S}{W_{//}} = \frac{c' b \cdot \sec\beta + N \cdot \tan\phi'}{W \cdot \sin\beta} = \frac{c' b \cdot \sec\beta + W \cdot \cos\beta \cdot \tan\phi'}{W \cdot \sin\beta}$$

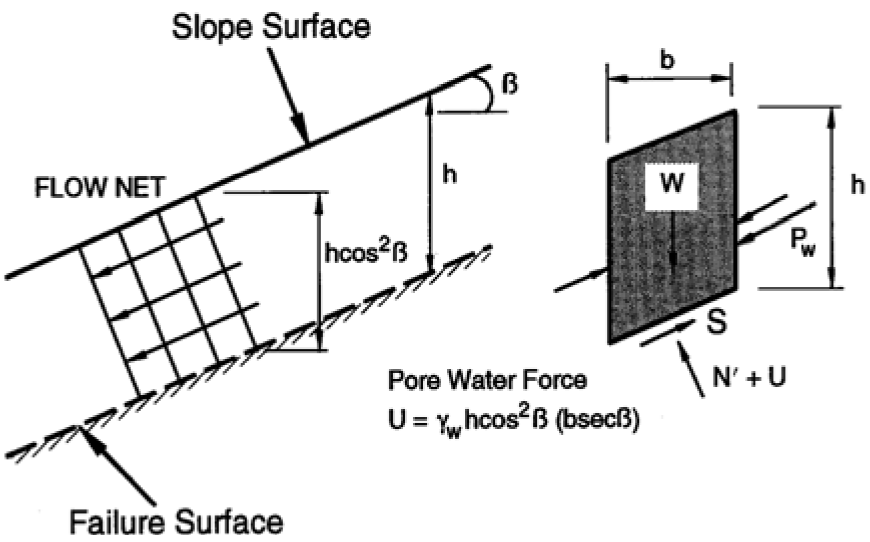
If we assume $c' = 0$, FOS will be:

$$FOS = \frac{\tan\phi}{\tan\beta}$$

INFINITE SLOPE ANALYSIS – Soil with seepage

Seepage parallel to the slope

If a saturated slope, in cohesive c - ϕ soil, has seepage parallel to the slope surface the same limit equilibrium concepts may be applied to determine the FOS, which will now depend on the effective normal force (N').



The total and effective weight of the slice, in this case, are respectively given by:

$$W = \gamma_{sat}\, b\, h\, 1 \qquad \text{and} \qquad W' = (\gamma_{sat} - \gamma_w)\, b\, h\, 1 = \gamma'\, b\, h\, 1$$

INFINITE SLOPE ANALYSIS – Soil with seepage

The effective normal force N' is given by: $N' = N - U$

where *N* is the total normal force and *U* the pore water force acting on the base of the slice:

$N = W_{\perp} = W \cos\beta = \gamma_{sat}\, b\, h \cos\beta$

$$U = \left(\gamma_w \cdot h \cdot \cos^2\beta\right) \cdot \frac{b}{\cos\beta} = \gamma_w \cdot b \cdot h \cdot \cos\beta$$

So the effective normal force N' is given by:

$$N' = (\gamma_{sat} - \gamma_w)\, b\, h\, 1 \cos\beta = \gamma'\, b\, h\, 1 \cos\beta$$

The driving forces can be given by:

$W'_{//} = W' \sin\beta = \gamma'\, b\, h\, 1 \sin\beta$

$P_w = p_w V = \gamma_w\, i\, V = \gamma_w\, i\, b\, h\, 1$

INFINITE SLOPE ANALYSIS – Soil with seepage

where P_w is the seepage force and i is the hydraulic gradient defined as the ratio between the hydraulic head measurements over the length of the flow path:

$$i = \frac{\Delta h}{L} = \frac{b \cdot \tan\beta}{b / \cos\beta} = \sin\beta$$

So P_w will be given by:

$$P_w = \gamma_w \, i \, b \, h \, 1 = \gamma_w \, b \, h \, 1 \sin\beta$$

The available frictional strength along the failure plane will depend on ϕ' and the effective normal force and is given by

$$S = c' \, b \sec\beta + (N - U) \tan\phi'$$

$$= c' \, b \sec\beta + N' \tan\phi' = c' \, b \sec\beta + \gamma' \, b \, h \, 1 \cos\beta \tan\phi'$$

INFINITE SLOPE ANALYSIS – Cohesionless Soil

So the FOS for this case will be

$$FOS = \frac{S}{W_{//} + P_w} = \frac{c' \cdot b \cdot \sec\beta + \gamma' \cdot b \cdot h \cdot \cos\beta \cdot \tan\phi}{\gamma' \cdot b \cdot h \cdot \sin\beta + \gamma_w \cdot b \cdot h \cdot \sin\beta} = \frac{c' \cdot b \cdot \sec\beta + \gamma' \cdot b \cdot h \cdot \cos\beta \cdot \tan\phi}{\gamma_{sat} \cdot b \cdot h \cdot \sin\beta}$$

For a $c = 0$ soil, the above expression may be simplified to give

$$FOS = \frac{\gamma'}{\gamma_{sat}} \cdot \frac{\tan\phi'}{\tan\beta}$$

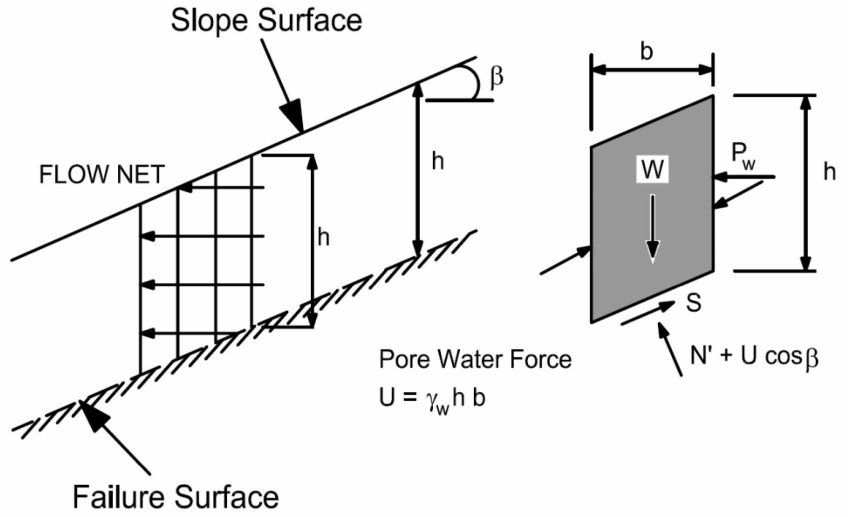
If we suppose $\gamma' \approx \gamma_w$, $\gamma_{sat} = \gamma' + \gamma_w \approx 2\gamma'$, so FOS become:

$$FOS = \frac{1}{2} \cdot \frac{\tan\phi'}{\tan\beta}$$

INFINITE SLOPE ANALYSIS – Soil with seepage

Horizontal Seepage

The total and effective weight of the slice, also in this case, are respectively given by:



$W = \gamma_{sat}\ b\ h\ 1$

$W' = (\gamma_{sat} - \gamma_w)\ b\ h\ 1 = \gamma'\ b\ h\ 1$

The effective normal force N' is given by: $N' = N - U \cos\beta$

where *N* is the total normal force and *U* the pore water force acting on the base of the slice:

$N = W_{\perp} = W \cos\beta = \gamma_{sat}\ b\ h \cos\beta$ and $U = \gamma_w\ b\ h$

INFINITE SLOPE ANALYSIS – Soil with seepage

So the effective normal force N' is given by:

$$N' = (\gamma_{sat} - \gamma_w)\, b\, h\, 1 \cos\beta = \gamma'\, b\, h\, 1 \cos\beta$$

The driving forces can be given by:

$W'_{//} = W' \sin\beta = \gamma'\, b\, h\, 1 \sin\beta$

$P_w = p_w\, V = \gamma_w\, i\, V = \gamma_w\, i\, b\, h\, 1$

where P_w is the seepage force and *i* is the hydraulic gradient defined as the ratio between the hydraulic head measurements over the length of the flow path, that, in this case, is given by:

$$i = \frac{\Delta h}{L} = \frac{b \cdot \tan\beta}{b} = \tan\beta$$

INFINITE SLOPE ANALYSIS – Soil with seepage

So P_w will be given by:

$$P_w = \gamma_w \, i \, b \, h \, 1 = \gamma_w \, b \, h \, 1 \tan\beta$$

In this seepage condition, P_w lies on horizontal direction, so we can subdivide P_w in the perpendicular and parallel components:

$P_{w,\perp} = P_w \sin\beta = \gamma_w \, b \, h \, 1 \tan\beta \sin\beta$

$P_{w,//} = P_w \cos\beta = \gamma_w \, b \, h \, 1 \tan\beta \cos\beta$

The available frictional strength along the failure plane will depend on ϕ', the effective normal force N' and $P_{w,\perp}$ and is given by

$$S = c' \, b \sec\beta + (N' - P_{w,\perp}) \tan \phi'$$

$$= c' \, b \sec\beta + (\gamma' \, b \, h \, 1 \cos\beta - \gamma_w \, b \, h \, 1 \tan\beta \sin\beta) \tan \phi'$$

INFINITE SLOPE ANALYSIS – Cohesionless Soil

Then, the FOS for this case will be

$$FOS = \frac{S}{W_{//} + P_{w//}} = \frac{c' b \sec\beta + (\gamma' b\, h \cos\beta - \gamma_w\, b\, h \tan\beta \sin\beta) \tan\phi'}{\gamma' b\, h \sin\beta + \gamma_w\, b\, h \tan\beta \cos\beta}$$

If we suppose *c' = 0* and $\gamma' \approx \gamma_w$ FOS become:

$$FOS = \frac{(\cos\beta - \tan\beta \sin\beta) \tan\phi'}{(\sin\beta + \tan\beta \cos\beta)} = \frac{\cos\beta\,(1 - \tan^2\beta) \tan\phi'}{2 \sin\beta}$$

$$= \frac{(1 - \tan^2\beta) \tan\phi'}{2 \tan\beta} = \frac{\tan\phi'}{\tan 2\beta}$$

INFINITE SLOPE ANALYSIS – Comparison

An example can show better the differences of the previous determined FOSs.

Example:

Cohesionless soil: γ = 18 kN/m3, f' = 30°, FOS_{min} = 1,3.

Calculate maximum slope angles in all three previous slope configurations.

Dry Soil

$$\beta_{MAX} = \arctan\left(\frac{\tan\phi'}{FOS}\right) = \arctan\left(\frac{\tan 30°}{1{,}3}\right) = 24°$$

Seepage parallel to the slope

$$\beta_{MAX} = \arctan\left(\frac{\tan\phi'}{2\cdot FOS}\right) = \arctan\left(\frac{\tan 30°}{2\cdot 1{,}3}\right) = 12{,}5°$$

Horizontal Seepage

$$\beta_{MAX} = \frac{1}{2}\arctan\left(\frac{\tan\phi'}{FOS}\right) = \arctan\frac{1}{2}\left(\frac{\tan 30°}{1{,}3}\right) = 12°$$

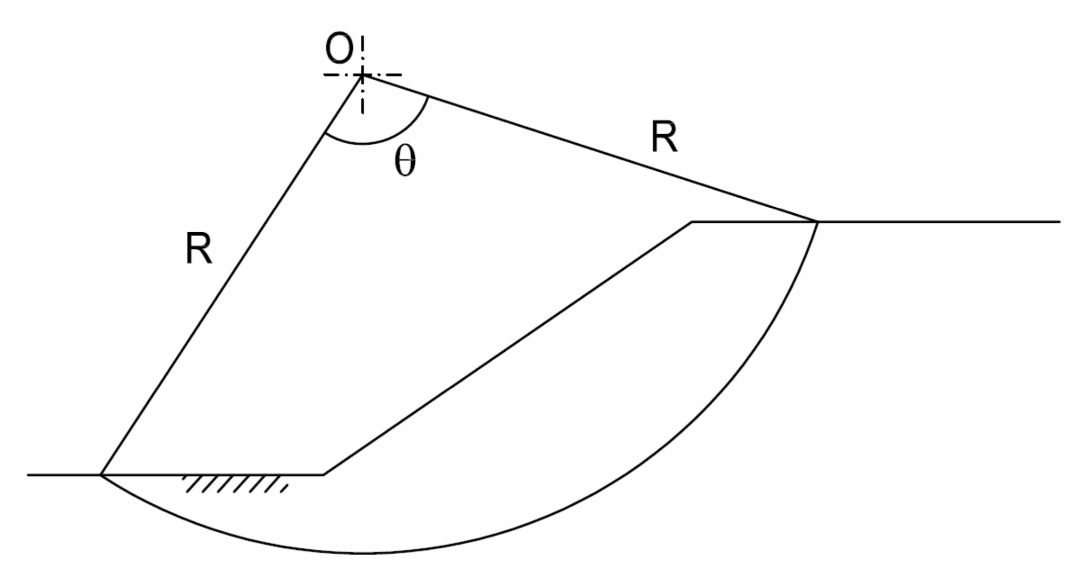
It is possible to note that the most onerous configuration is that of horizontal seepage.

CIRCULAR SURFACE ANALYSIS

Circular failure surfaces are found to be the most critical in slopes consisting of homogeneous materials.

There are two analytical methods that may be used to calculate the FOS for a slope :

- the circular arc (ϕ = 0)
- Friction circle.

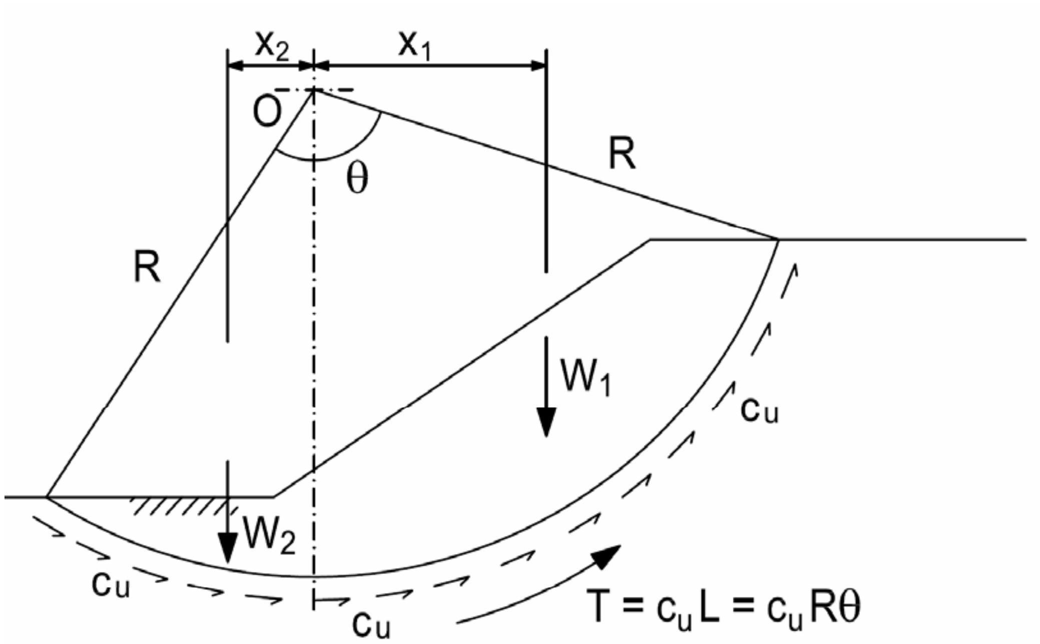


CIRCULAR SURFACE ANALYSIS – Circular Arc

The simplest circular analysis is based on the assumption that a rigid, cylindrical block will fail by rotation about its center and that the shear strength along the failure surface is defined by the undrained strength.

As the undrained strength is used, the angle of internal friction, ϕ, is assumed to be zero (hence the *ϕ = 0 method*)

The FOS for such a slope may be analyzed by taking the ratio of the resisting and overturning moments about the center of the circular surface O.

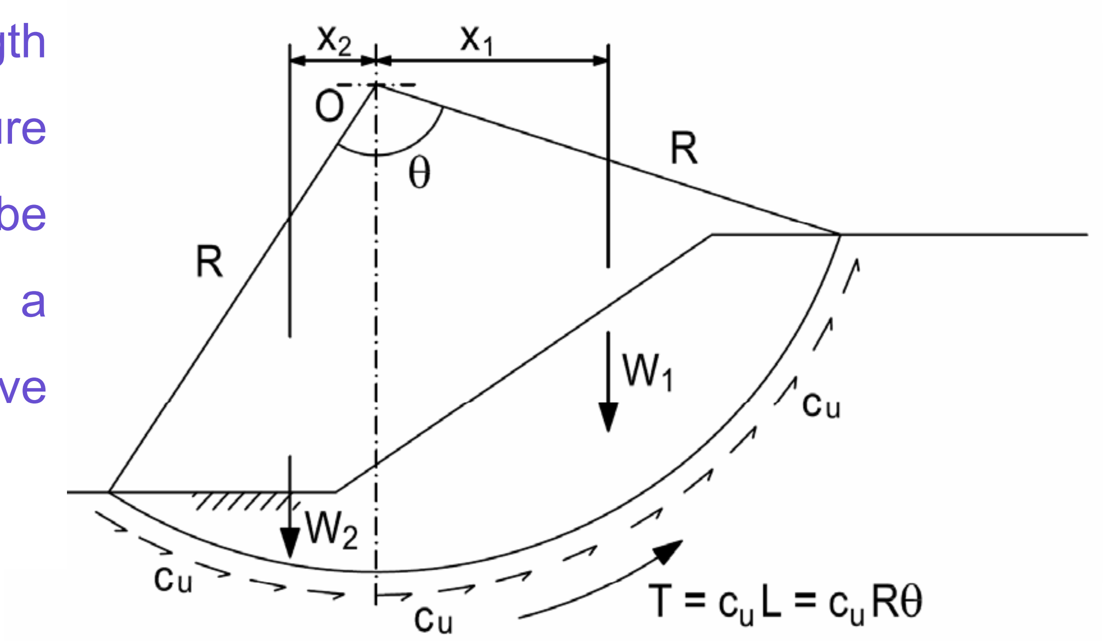


CIRCULAR SURFACE ANALYSIS – Circular Arc

If the overturning moments are given by W_1x_1 and resisting moments are given by $c_uLR + W_2x_2$, the factor of safety for the slope will be:

$$FOS = \frac{W_2x_2 + c_uLR}{W_1x_1}$$

If the undrained shear strength varies along the failure surface, the c_uL term must be modified and treated as a variable in the above formulation.



CIRCULAR SURFACE ANALYSIS – Friction Circle

This method is useful for homogeneous soils with $\phi > 0$, such that the shear strength depends on the normal stress.

It may be used when both cohesive and frictional components for shear strength have to be considered in the calculations.

The method is equally suitable for total or effective stress types of analysis.

The method attempts to satisfy the requirement of complete equilibrium by assuming the direction of the resultant of the normal and frictional component of strength mobilized along the failure surface. This direction corresponds to a line that forms a tangent to the friction circle, with a radius, $R_f = R \sin \phi_m$.

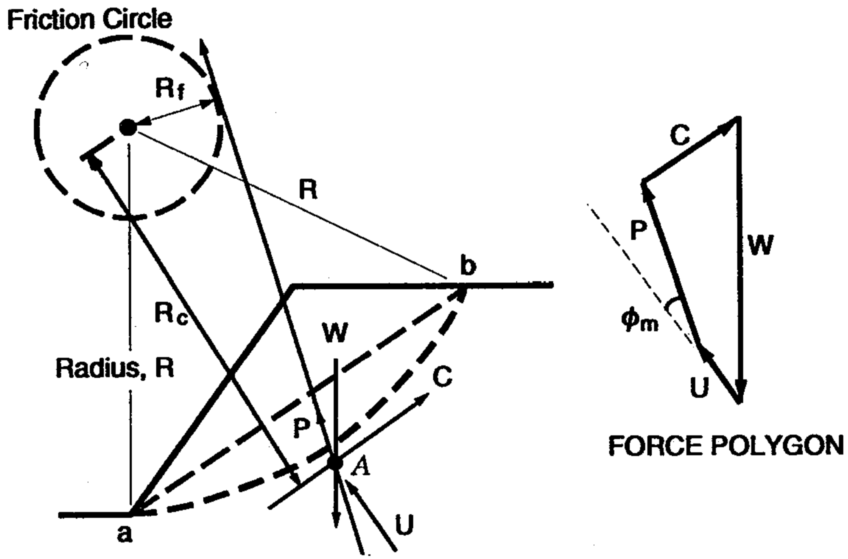
This assumption is guaranteed to give a lower bound FOS value.

CIRCULAR SURFACE ANALYSIS – Friction Circle

The cohesive shear stresses along the base of the *failure surface ab*, will have a resultant, C_m, that acts parallel to the direction of the *chord ab*.

Its location may be found by taking moments of the distribution and the resultant, C_m, about the circle center. This line of action of resultant, C_m, can be located using

$$\left(\frac{c}{F_c}\cdot \widehat{ab}\right)R = C_m \cdot R_c = \left(\frac{c}{F_c}\cdot \overline{ab}\right)R_c$$



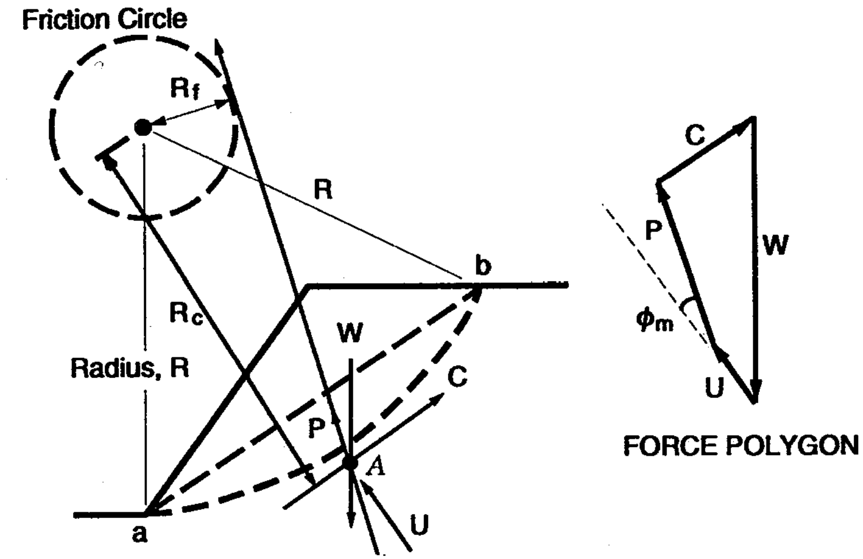
The distance R_c of this line from the center of the circle O is:

$$R_c = \frac{L_{arc}}{L_{chord}}\cdot R$$

CIRCULAR SURFACE ANALYSIS – Friction Circle

The actual point of application, A, is located at the intersection of the *effective* weight force, which is the resultant of the weight and any pore water forces. The resultant of the normal and frictional (shear) force, *P*, will then be inclined parallel to a line formed by a point of tangency to the friction circle and point A.

As the direction of C_m is known, the force polygon can be closed to obtain the value of the mobilized cohesive force. Again, the final FOS is computed with the assumption $F_\phi = F_c =$ FOS along the failure circular surface.



CIRCULAR SURFACE ANALYSIS – Friction Circle

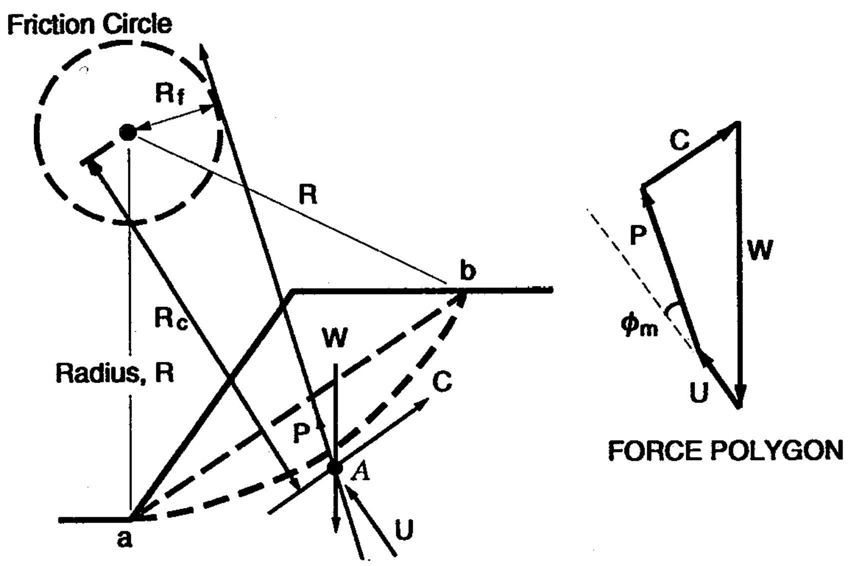
The solution procedure is usually followed graphically. Solution procedure:

(1) Calculate weight of slide, W.

(2) Calculate magnitude and direction of the resultant pore water force, U (may need to discretize slide into slices).

(3) Calculate perpendicular distance to the line of action of C_m.

(4) Find effective weight resultant, W ', from forces W and U, and its intersection with the line of action of C_m at A.



(5) Assume a value of $F\phi$.

(6) Calculate the mobilized friction angle $\phi_m = \tan^{-1} (\tan \phi) / F_\phi$.

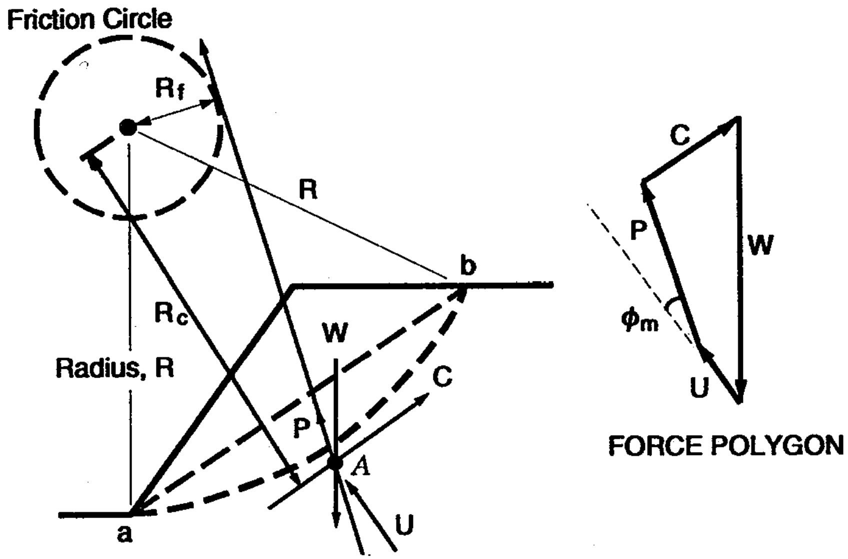
CIRCULAR SURFACE ANALYSIS – Friction Circle

(7) Draw the friction circle, with radius R_f = R sin ϕ_m.

(8) Draw the force polygon with W ' appropriately inclined, and passing through point A.

(9) Draw the direction of P, tangential to the friction circle.

(10) Draw direction of C_m, according to the inclination of the chord linking the end-points of the failure surface.



(11) The closed polygon will then provide the value of C_m.

(12) Using this value of C_m, calculate

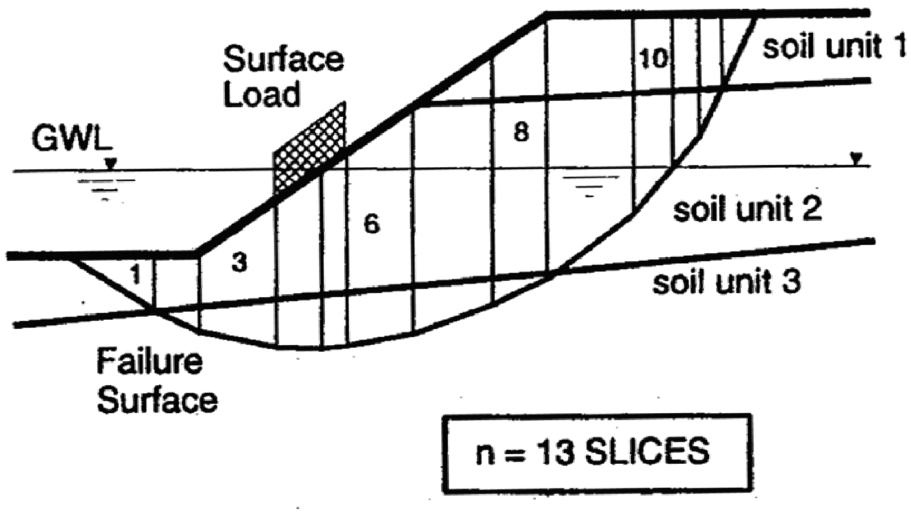
$$F_c = c\, L_{chord} / C_m.$$

(13) Repeat steps 5 to 12 until $F_c \approx F_\phi$.

METHOD OF SLICES

If the mobilized strength for a c – ϕ soil is to be calculated, the distribution of the effective normal stresses along the failure surface must be known. This condition is usually analyzed by discretizing the mass of the failure slope into smaller slices and treating each individual slice as a unique sliding block.

The *method of slices* is used by most computer programs, as it can readily accommodate complex slope geometries, variable soil conditions, and the influence of external boundary loads.



METHOD OF SLICES – Basic assumptions (1)

There are several formulations of the method of slices in relation to the assumption that the numerous authors had made. We can subdivide them in three categories:

1. Assumptions on interslice forces direction
   (Bishop, 1955; Spencer, 1967; Morgenstern-Price, 1965)

2. Assumptions on the thrust line position (Janbu, 1954)

3. Assumptions on the interslice forces distribution
   (Sarma, 1973; Correia, 1988)

METHOD OF SLICES – Basic assumptions (2)

All these methods are based on similar concepts, but they give different FOSs values through the previous assumptions. However, all of them are based on the *Mohr-Coulomb failure criterion* (rigid-perfect plastic behavior).

All limit equilibrium methods for slope stability analysis divide a slide mass into *n* smaller slices, so that we can approximate the irregular base of the slice (often an arc) as a chord.

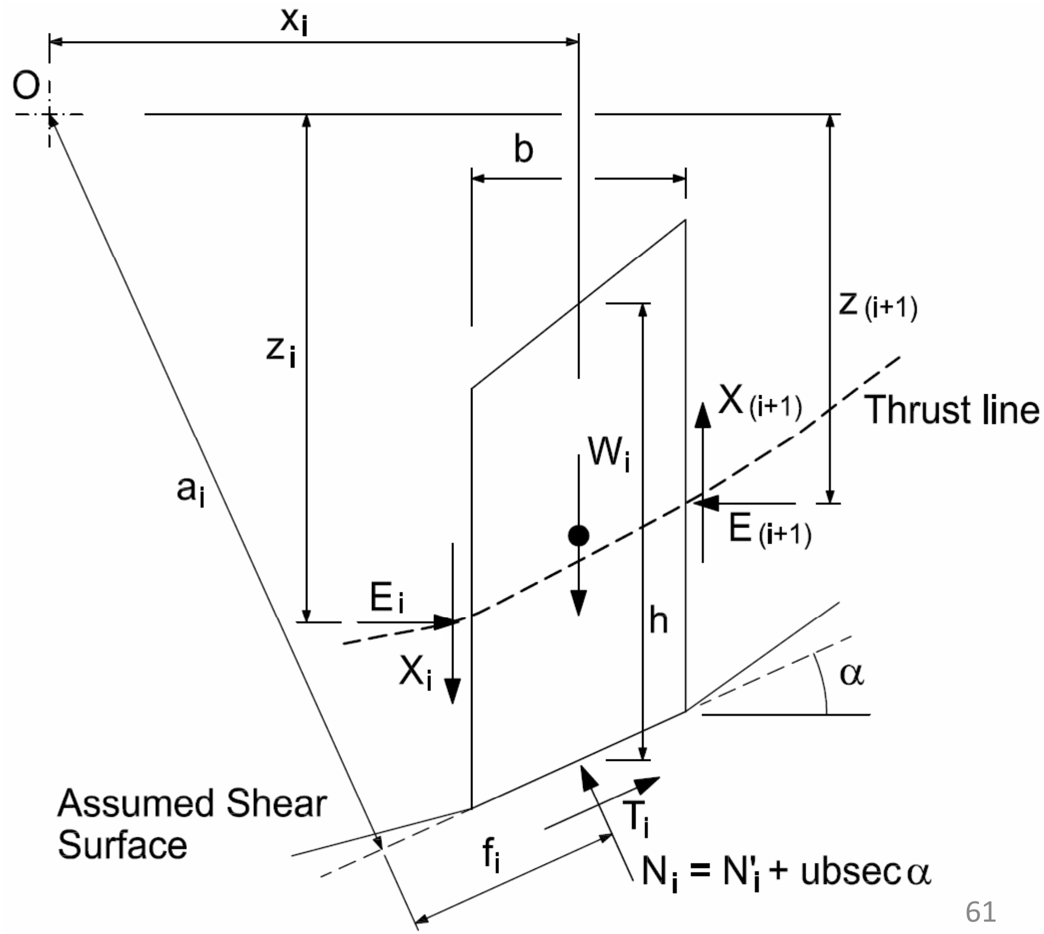
Another hypothesis is that FOS is considered constant along the failure surface.

METHOD OF SLICES – System of forces

Each slice is affected by a general system of forces.

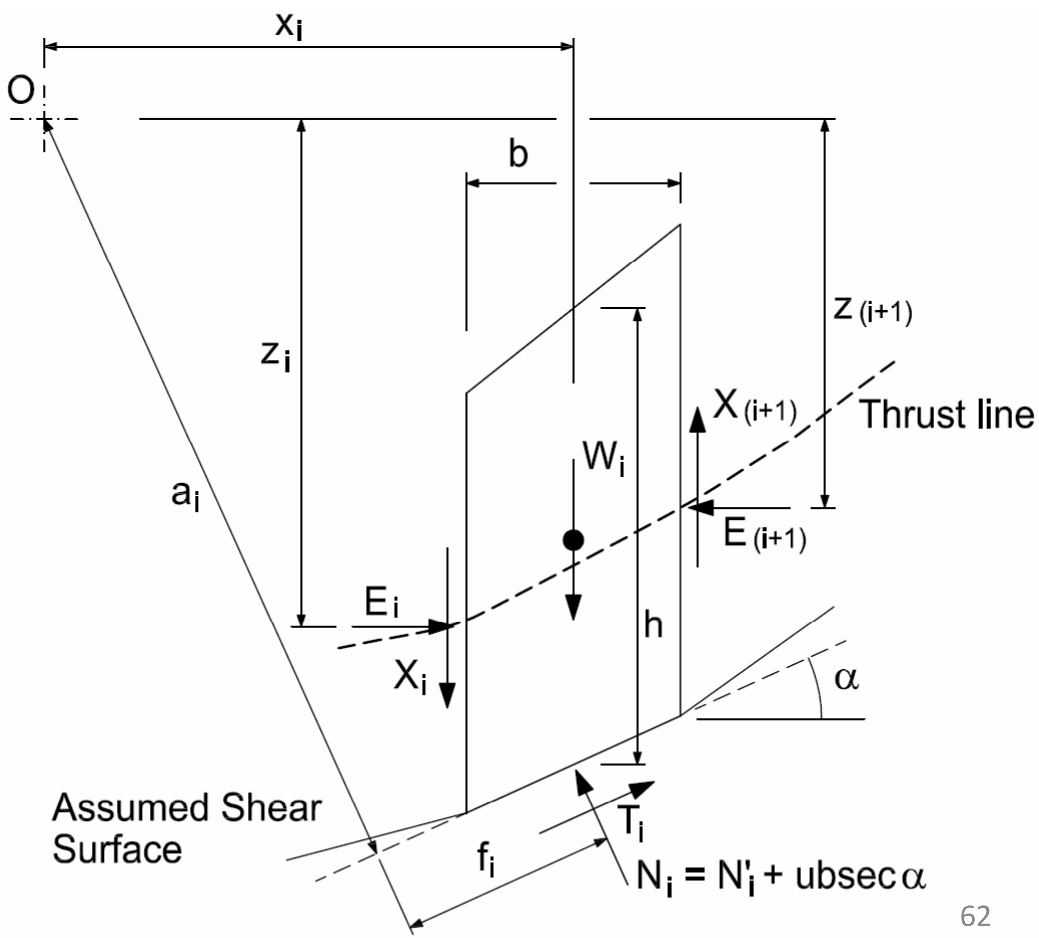
The *thrust line* connects the points of application of the interslice forces.

The location of this thrust line may be assumed or its location may be determined using a rigorous method of analysis that satisfies complete equilibrium.



METHOD OF SLICES – System of forces

The popular simplified methods of analysis neglect the location of the interslice force because complete equilibrium is not satisfied for the failure mass.



For this system, there are *(6n – 2) unknowns*. Also, since only four equations for each slice (*4n equations*) can be written for the limit equilibrium for the system, the solution is statically indeterminate.

METHOD OF SLICES – System of forces

Equations	Condition
n	Moment equilibrium for each slice
2n	Force equilibrium in two directions (for each slice)
n	Mohr – Coulomb relationship between shear strength and formal effective stress
4n	Total number of equations

Unknowns	Variable
1	FOS
n	Normal force at base of each slice, N'_i
n	Location of normal force, N'_i
n	Shear force at base of each slice, T_i
n - 1	Interslice horizontal force, E_i
n - 1	Location of interslice horizontal force, E_i
n - 1	Interslice vertical force, X_i
6n - 2	Total number of unknowns

METHOD OF SLICES – Assumptions (3)

A solution is possible providing the number of unknowns can be reduced by making some simplifying assumptions.

One of these common assumptions is that the normal forces on the base of the slices acts at the midpoint, thus reducing the number of unknowns to (5n – 2).

This then requires an additional (n – 2) assumptions to make the problem determinate.

It is these assumptions that generally categorize the available methods of analysis.

METHOD OF SLICES – Common Methods

List of the common methods of analysis and the conditions of static equilibrium that are satisfied in determining the FOS.

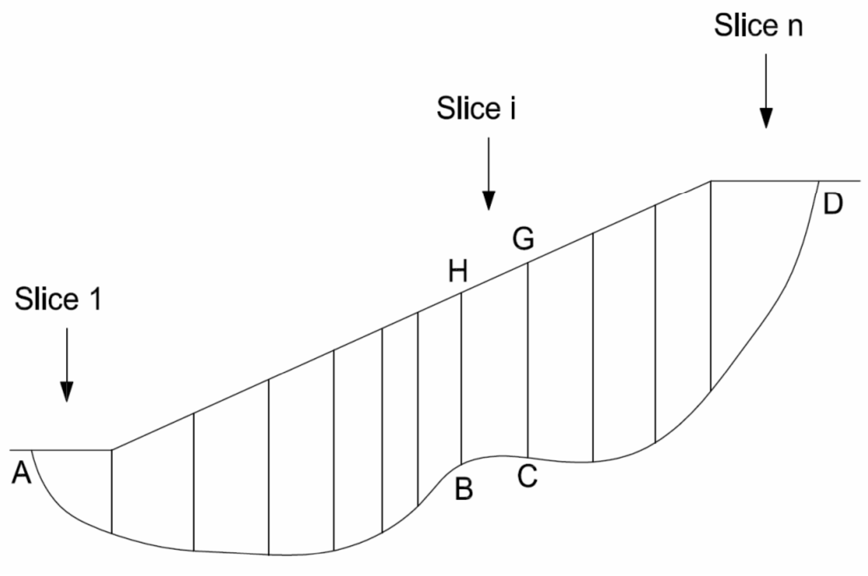
Method	Force - Equilibrium		Moment Equilibrium
	x	y	
Ordinary method of slices (OMS)	No	No	Yes
Bishop's simplified	Yes	No	Yes
Janbu's simplified	Yes	Yes	No
Corps of Engineers	Yes	Yes	No
Lowe and Karafiath	Yes	Yes	No
Janbu's generalized	Yes	Yes	No
Bishop's rigorous	Yes	Yes	Yes
Spencer's	Yes	Yes	Yes
Sarma's	Yes	Yes	Yes
Morgenstern – Price	Yes	Yes	Yes

METHOD OF SLICES – General formulation

Consider a body of soil on the point of sliding on the surface ABCD. For the purpose of analysis, we will divide the whole body of soil abode the surface of sliding into *n elementary slices*, separated by *n-1 vertical boundaries*.

The choice of vertical interslice boundaries is merely a matter of convenience.

In general, the failure condition is not satisfied on these surfaces.



METHOD OF SLICES – General formulation

If the body is stable, force and moment equilibrium conditions must be satisfied for each slice, and also for the whole body.

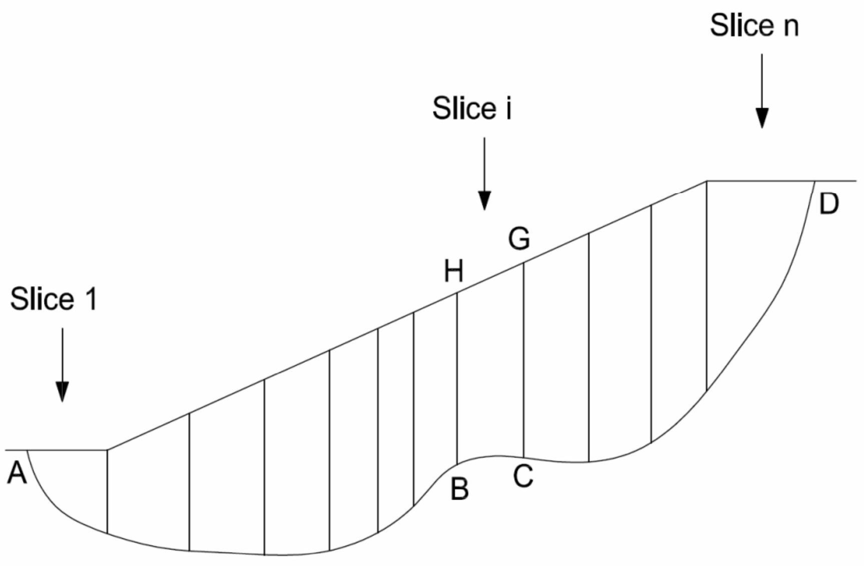
The failure condition

$$\tau_f = c' + (\sigma_n - u) \tan \phi'$$

must be satisfied everywhere on the surface ABCD.

If $F_c = F_\phi = F$ along the failure surface, we may define a factor of safety in the form

$$\tau = \frac{\tau_f}{F} = \frac{c'}{F_c} + (\sigma_n - u)\frac{\tan \phi'}{F_\phi}$$



METHOD OF SLICES – General formulation

For the slice i: $T_i = [\tau\ b\ \sec\alpha]_i$; $N_i = [\sigma_n\ b\ \sec\alpha]_i$

Then:

$$T_i = [\tau\ b\ \sec\alpha]_i = \frac{1}{F}\left[c'b\ \sec\alpha + (N - ub\ \sec\alpha)\tan\phi'\right]_i$$

It will be convenient to specify the pore pressure u_i on BC in term of the pore pressure ratio r_u equal to $[ub/W]_i$ where W_i is the weight of slice i. Then

$$T_i = \frac{1}{F}\left[c'b\ \sec\alpha + (N - Wr_u\ \sec\alpha)\tan\phi'\right]_i \qquad (1)$$

Vertical forces equilibrium of slice i

$$[T\sin\alpha + N\cos\alpha]_i = [W - \Delta X]_i \qquad (2)$$

where

$$\Delta X_i = X_{(i+1)} - X_i$$

METHOD OF SLICES – General formulation

Substituting equation (1) into number (2) we obtain

$$\left[\frac{\sin\alpha}{F}\left\{c'b\sec\alpha + (N - Wr_u\sec\alpha)\tan\phi'\right\} + N\cos\alpha\right]_i = [W - \Delta X]_i$$

$$\left[N\left(\frac{\sin\alpha}{F}\tan\phi' + \cos\alpha\right) + c'b\frac{\sin\alpha}{F}\sec\alpha - \frac{\sin\alpha}{F}\left(Wr_u\sec\alpha\tan\phi'\right)\right]_i = [W - \Delta X]_i$$

Rearranging the terms, this yields

$$N_i = \frac{1}{m_{\alpha,i}}\left[W\left(1 + r_u\tan\alpha\frac{\tan\phi'}{F}\right) - \frac{c'b}{F}\tan\alpha - \Delta X\right]_i \qquad (3)$$

where

$$m_{\alpha,i} = \left[\cos\alpha + \sin\alpha\frac{\tan\phi'}{F}\right]_i$$

Substitute equation (3) into number (1) to give

$$T_i = \frac{1}{F\,m_{\alpha,i}}\left[c'b + \{W(1 - r_u) - \Delta X\}\tan\phi'\right]_i \qquad (4)$$

METHOD OF SLICES – General formulation

Tangential forces equilibrium of slice i

$$T_i + \Delta E_i \cos\alpha_i + \Delta X_i \sin\alpha_i - W_i \sin\alpha_i = 0 \qquad (5)$$

Rearranging the terms, this yields

$$\frac{T_i}{\cos\alpha_i} - (W_i - \Delta X_i)\tan\alpha_i = \Delta E_i$$

Substituting T_i with equation (4) we obtain

$$\frac{\sec\alpha_i}{F\, m_{\alpha,i}}\left[c'b + \{W(1-r_u) - \Delta X\}\tan\phi'\right]_i - \left[(W - \Delta X)\tan\alpha\right]_i = \Delta E_i$$

In considering the equilibrium of the whole soil mass, the internal interslice forces (E_2 to E_n and X_2 to X_n) must vanish. Also if there are no external forces on the end slices,

$$E_1 = E_{(n+1)} = X_1 = X_{(n+1)} = 0$$

METHOD OF SLICES – General formulation

So that

$$\frac{1}{F}\sum_{i=1}^{n}\frac{\sec\alpha_i}{m_{\alpha,i}}\left[c'b + \{W(1-r_u) - \Delta X\}\tan\phi'\right]_i - \sum_{i=1}^{n}\left[(W - \Delta X)\tan\alpha\right]_i = 0 \quad (6)$$

Moment equilibrium of slice i around pivot O

$$W_i x_i + X_i\left(x_i - \frac{b_i}{2}\right) - X_{i+1}\left(x_i + \frac{b_i}{2}\right) - E_i z_i + E_{i+1} z_{i+1} = T_i a_i + N_i f_i$$

Since the moments of all internal forces (E_i, X_i) must also vanish when considering the whole body,

$$\sum_{i=1}^{n} T_i a_i = \sum_{i=1}^{n}(W_i x_i - N_i f_i) \quad (7)$$

METHOD OF SLICES – General formulation

Substituting the expression for T_i and N_i obtained previously this becomes

$$F = \frac{\sum_{i=1}^{n} \frac{a_i}{m_{\alpha,i}} \left[c'b + \{W(1 - r_u) - \Delta X\} \tan \phi'\right]_i}{\sum_{i=1}^{n} [W\, x]_i - \sum_{i=1}^{n} \frac{f_i}{m_{\alpha,i}} \left[W \left(1 + r_u \tan \alpha \frac{\tan \phi'}{F}\right) - \frac{c'b}{F} \tan \alpha - \Delta X\right]_i} \qquad (8)$$

In the case we have circular failure surface, equation (8) become

$$F = \frac{\sum_{i=1}^{n} \frac{1}{m_{\alpha,i}} \left[c'b + \{W(1 - r_u) - \Delta X\} \tan \phi'\right]_i}{\sum_{i=1}^{n} [W \sin \alpha]_i} \qquad (9)$$

METHOD OF SLICES – OMS

The *Ordinary Method of Slices (OMS)*: this method (Fellenius, 1927, 1936) is one of the simplest procedures based on the method of slices and neglects all interslice forces (X_i and E_i) and fails to satisfy force equilibrium for the slide mass as well as for individual slices. So the number of unknowns become: $(5n - 2) - \underbrace{(n - 1)}_{X_i} - \underbrace{(n - 1)}_{E_i} - \underbrace{(n - 1)}_{z_i} = 2n + 1$ unknowns < 4n equations.

This method consider a circular failure surface.

If we remember that:

$$T_i = \frac{1}{F}\left[c' b \sec \alpha + (N - ub \sec \alpha) \tan \phi'\right]_i$$

and manipulating equation (7) to give

$$\sum_{i=1}^{n} T_i R = \sum_{i=1}^{n} (W_i R \sin \alpha_i)$$

METHOD OF SLICES – OMS

we obtain

$$F = \frac{\sum_{i=1}^{n} \left[c' b \sec\alpha + (N - u\, b \sec\alpha) \tan\phi'\right]_i}{\sum_{i=1}^{n} \left[W \sin\alpha\right]_i}$$

The Ordinary Method of Slices can be used also in the case of layered soils. It is sufficient to consider the correct soil strength parameter (c' and ϕ') at the base of each slice.

METHOD OF SLICES – Bishop's Method

The *Bishop's Method* is one of the most famous methods based on limit equilibrium. He makes the hypothesis of a circular failure surface.

There are two different approaches:

a. *Bishop's Rigorous Method.*

   Satisfy moment equilibrium – circular failure surface

b. *Bishop's Simplified Method.*

   Satisfy complete equilibrium – circular failure surface

METHOD OF SLICES – Bishop's Method

a. Bishop's Rigorous Method

Bishop (1955) considers in his rigorous formulation equation (6) and equation (9).

He also imposed that

$$X_i = \lambda \, f(x)$$

where λ is a constant unknown and f(x) a known function.

We saw early that there are n – 2 additional unknown which make the problem indeterminate. In this case, due to the previous assumption, all X_i become known if λ is defined. Therefore, the total number of unknowns is: (5n – 2) – (n – 1) + 1 = 4n unknowns = 4n equation. The problem is determinate and the equilibrium equations are satisfied.

METHOD OF SLICES – Bishop's Method

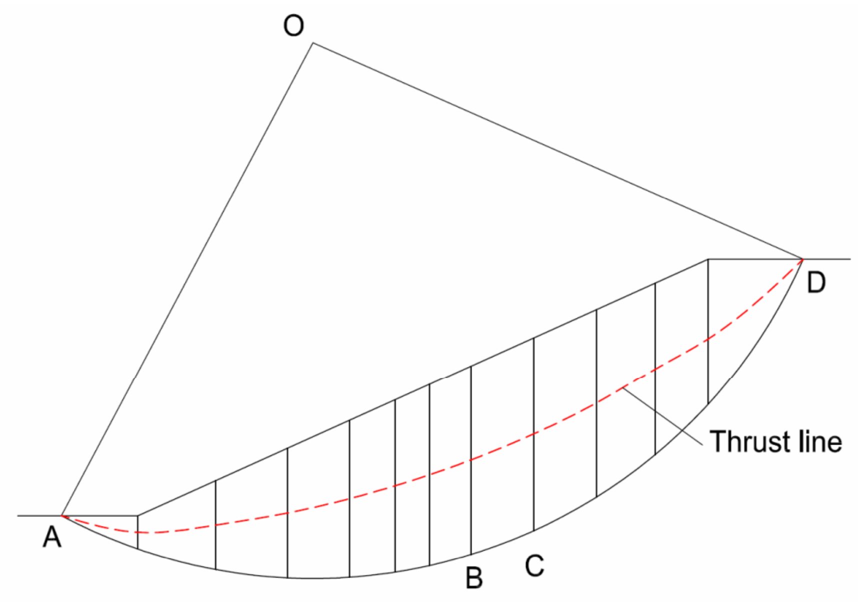
Note: There could be several combinations of the unknown λ and the assumed known function f(x) that satisfy the problem. However, some of these combinations could be not admissible. It is important to check out other two requirements:

1. *Shear strength along vertical interslice surfaces must be less than the failure strength of the soil*, i.e.

   $$\tau_i = X_i / A_i < \tau_f = c' + \sigma' \tan \phi'$$

   In fact, these surfaces can't reach the failure condition.

2. *The thrust line must be located inside the sliding mass*.



METHOD OF SLICES – Bishop's Method

b. Bishop's Simplified Method

In his simplified approach, Bishop (1955) assumes that all vertical interslice shear forces X_i are zero, reducing the number of unknowns by (n – 1). This leaves (4n – 1) unknowns, leaving the solution overdetermined.

So, $X_i = 0$, $\lambda = 0$ and $\Delta X_i = 0$ give

$$F = \frac{\sum_{i=1}^{n} \frac{1}{m_{\alpha,i}} \left[c'b + \{W(1 - r_u)\} \tan \phi'\right]_i}{\sum_{i=1}^{n} [W \sin \alpha]_i}$$

METHOD OF SLICES – Bishop's Method

Solution procedure:

1) Assume initial value of F_0

2) Calculate with F_0 $m_{\alpha,i}$ for each slice

3) Determine F with the above expression F_0

   If $F = F_0$: stop procedure

   If $F \neq F_0$: iterate procedure – repeat steps (1) to (3)

METHOD OF SLICES – Janbu's Method

Similar to Bishop's method, the *Janbu's method* can be discerned in two different approaches:

a. *Janbu's Simplified Method.*

   Satisfy force equilibrium – Any kind of failure surface

b. *Janbu's Generalized Method.*

   Janbu assumes a location of the thrust line, thereby reducing the number of unknowns to (4n – 1). Similar to the rigorous Bishop method, Janbu also suggests that the actual location of the thrust line is an additional unknown, and thus equilibrium can be satisfied rigorously if the assumption selects the correct thrust line. – Any kind of failure surface

METHOD OF SLICES – Janbu's Method

a. Janbu's Simplified Method

Janbu (1954, 1957, 1973) assumes zero interslice vertical shear forces X_i, reducing the number of unknowns to (4n – 1). This leads to an overdetermined solution that will not satisfy moment equilibrium conditions.

Equation (6) can be rewritten

$$\frac{1}{F}\sum_{i=1}^{n}\frac{\sec\alpha_i}{m_{\alpha,i}}\left[c'b+\{W(1-r_u)\}\tan\phi'\right]_i-\sum_{i=1}^{n}\left[W\tan\alpha\right]_i=0$$

From which:

$$F=\frac{\sum_{i=1}^{n}\frac{\sec\alpha_i}{m_{\alpha,i}}\left[c'b+\{W(1-r_u)\}\tan\phi'\right]_i}{\sum_{i=1}^{n}\left[W\tan\alpha\right]_i}$$

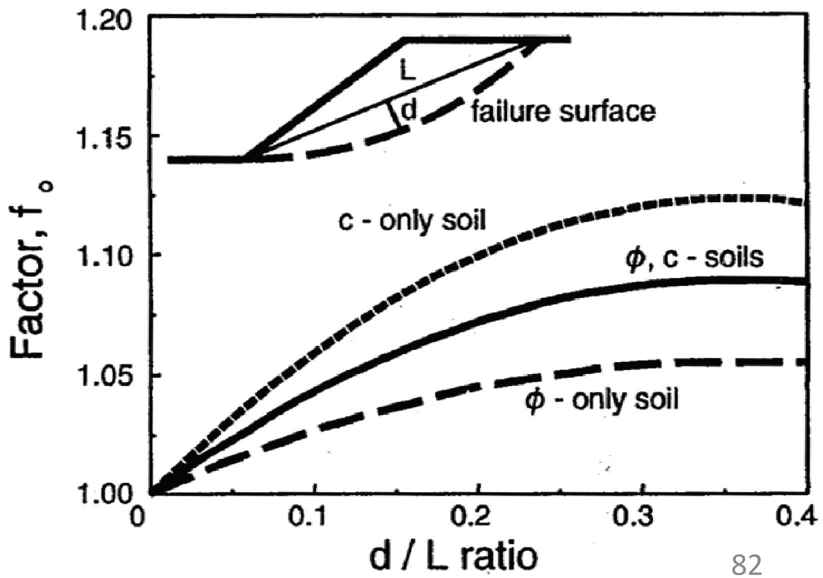
METHOD OF SLICES – Janbu's Method

The assumption of zero vertical interslice forces give an overvalued FOS. To account for this inadequacy, Janbu presented a correction factor, f_0 (>1), so that:

$$F_{Janbu} = f_0 \cdot F_{calculated}$$

This modification factor is a function of the slide geometry and the strength parameters of the soil.

There is no consensus concerning the selection of the appropriate f_0 value for a surface intersecting different soil types. In cases where such a mixed variety of soils is present, the $c - \phi$ curve is generally used to correct the calculated FOS value.



METHOD OF SLICES – Janbu's Method

For convenience, this modification factor can also be calculated according to the formula

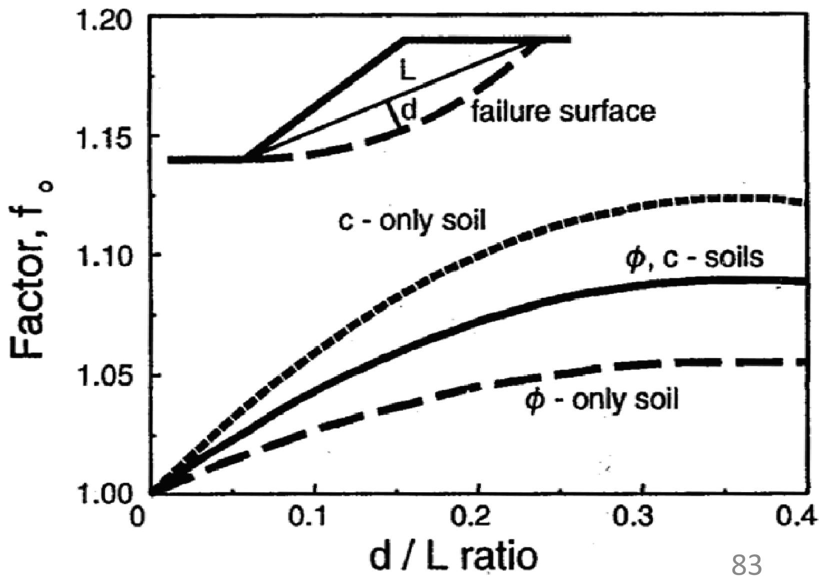
$$f_0 = 1 + b_1 \left[\frac{d}{L} - 1.4\left(\frac{d}{L}\right)^2\right]$$

where b_1 varies according to the soil type:

c only soils: $b_1 = 0.69$

ϕ only soils: $b_1 = 0.31$

c and ϕ soils: $b_1 = 0.5$



METHOD OF SLICES – Advanced Methods

Morgenstern – Price Method Morgenstern and Price (1965) propose a method in which the inclination of the interslice resultant force is assumed to vary according to a “portion” of an arbitrary function, i.e.

$$X_i / E_i = \lambda f(x)$$

This additional “portion” of a selected function introduces an additional unknown, leaving 4n unknowns and 4n equations.

The Morgenstern – Price Method satisfy equations (6) and (8) and then the force and moment equilibrium for any kind of failure surface.

Spencer’s Method Spencer (1967, 1973) rigorously satisfies static equilibrium by assuming that the resultant interslice force has a constant, but unknown, inclination. This method derives from Morgenstern – Price Method, assuming f(x) = cost.

METHOD OF SLICES – GLE

A *general limit equilibrium* (GLE) formulation (Chugh, 1986; Fredlund et al., 1981) can be developed to encompass most of the assumptions used by the various methods and may be used to analyze circular and noncircular failure surfaces.

In view of this universal applicability, the GLE formulation has become one of the most popular methods as its generalization offers the ability to model a discrete version of the Morgenstern and Price (1965) procedure via the function used to describe the distribution of the interslice force angles.

The method can be used to satisfy either force and moment equilibrium or, if required, just the force equilibrium conditions.

METHOD OF SLICES – GLE

The GLE procedure relies on the selection of an appropriate function that describes the variation of the interslice force angles to satisfy complete equilibrium.

The main difficulty in using the GLE procedure is related to the requirement that the user verify the reliability and “reasonableness” of the calculated FOS. This additional complexity prevents the general use of the GLE method for automatic search procedures that attempt to identify the critical failure surface. However, single failure surfaces can be analyzed and the detailed solution examined for reasonableness.

STABILITY CHARTS

Slope stability charts are useful for preliminary analysis, to compare alternates that can later be examined by more detailed analyses.

Chart solutions also provide a rapid means of checking the results of detailed analyses.

Another use for slope stability charts is to back-calculate strength values for failed slopes to aid in planning remedial measures. This can be done by assuming an FOS of unity for the conditions at failure and solving for the unknown shear strength.

The major shortcoming in using design charts is that most of them are for ideal, homogeneous soil conditions, which are not encountered in practice.

STABILITY CHARTS – historical background

Author	Parameters	Slope Inclinations	Analytical Methods	Notes
Taylor (1948)	c_u	0-90°	$\phi = 0$	Undrained analysis
	c, ϕ	0-90°	Friction circle	Dry slopes only
Bishop and Morgenstern (1960)	c, ϕ, r_u	11-26.5°	Bishop	One of the first to include effects of water
Gibson and Morgenstern (1960)	c_u	0-90°	$\phi = 0$	Undrained analysis with c_u increasing linearly with depth; zero strength at ground level
Spencer (1967)	c, ϕ, r_u	0-34°	Spencer	Toe circle only
Janbu (1968)	c_u	0-90°	$\phi = 0$	Extensive series of charts for seepage and tension crack effects
	c, ϕ, r_u		Janbu GPS	
Hunter and Schuster (1968)	c_u	0-90°	$\phi = 0$	Undrained analysis with c_u increasing linearly with depth; finite strength at ground level
Chen and Giger (1971)	c, ϕ	20-90°	Limit analysis	
O'Connor and Mitchell (1977)	c, ϕ, r_u	11-26°	Bishop	Extended Bishop and Morgenstern (1960) to include N_c = 0.1
Hoek and Bray (1977)	c, ϕ	0-90°	Friction circle	Includes groundwater and tension cracks
	c, ϕ	0-90°	Wedge	3-D analysis of wedge block
Cousins (1978)	c, ϕ, r_u	0-45°	Friction circle	Extension of Taylor (1948)
Charles and Soares (1984)	ϕ	26-63°	Bishop	Nonlinear Mohr-Coulomb failure envelope, $\tau = A(\sigma')^b$
Barnes (1991)	c, ϕ, r_u	11-63°	Bishop	Extension of Bishop and Morgenstern (1960); wider range of slope angles

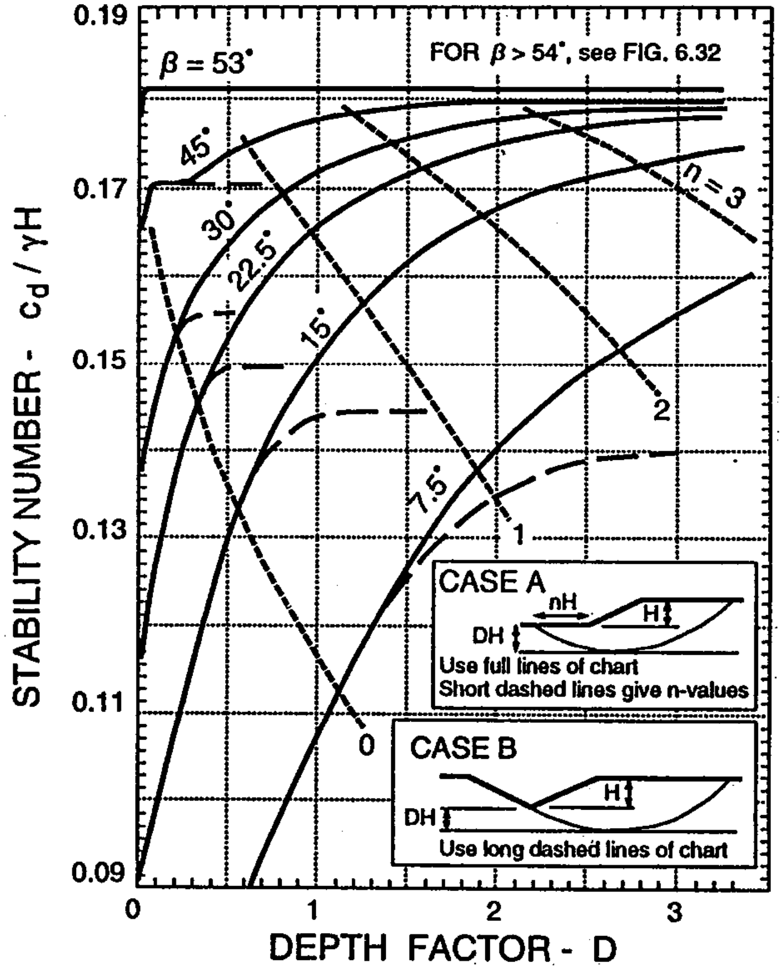
STABILITY CHARTS – Taylor's Charts

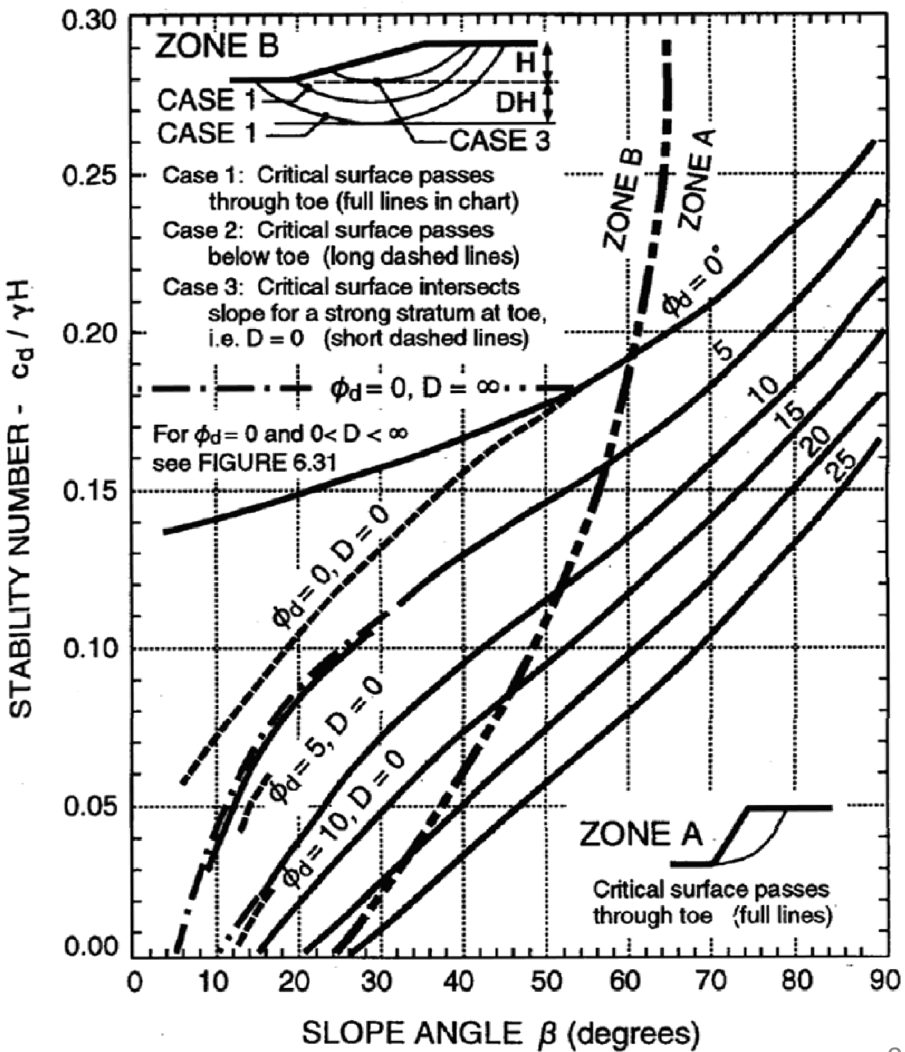
Taylor (1948) developed slope stability charts for soils with $\phi = 0$ and $\phi > 0$. As shown in these charts, the slope has an angle β, a height H, and base stratum at a depth of $D \cdot H$ below the toe, where D is a depth ratio.

The charts can be used to determine the developed cohesion, c_d, as shown by the solid curves, and $n \cdot H$, which is the distance from the toe to the failure circle, as indicated by the short dashed curve.

If there are loadings outside the toe that prevent the circle from passing below the toe, the long dashed curve should be used to determine the developed cohesion. Note that the solid and the long dashed curves converge as n approaches zero. The circle represented by the curves on the left of $n = 0$ do not pass below the toe, so the loading outside the toe has no influence on the developed cohesion.

STABILITY CHARTS – Taylor's Charts





STABILITY CHARTS – Bishop-Morgenstern's Charts

Bishop and Morgenstern's Procedure (1960) is quite more complex than Taylor's Charts. Compared to Taylor's method, this one, furthermore, can take into account water pore pressure inside the sliding mass and on the failure surface through water pore pressure ratio r_u:

$$r_u = \frac{u}{\sigma_v}$$

Bishop and Morgenstern's Procedure is suitable for effective stress analysis in homogeneous soils.

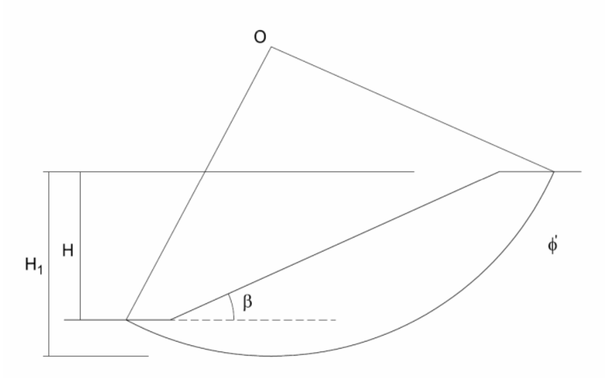
The charts are based on two parameters that are called stability factors *m* and *n*, so the FOS is defined as

$$FOS = m - n\, r_u$$

STABILITY CHARTS – Bishop-Morgenstern's Charts

m and *n* depend on several parameters:

- β, slope angle;
- ϕ', soil friction angle;
- $d_f = H_1/H$, depth factor
- $c' / (\gamma H)$, similar to Taylor's stability number



d_f has a small influence on the solution, so one can refer essentially to three value of it: 1.00, 1.25, 1.50.

In the charts are treated three conditions: $c' / (\gamma H) = 0.05$

$c' / (\gamma H) = 0.025$

$c' / (\gamma H) = 0$

In general, user's condition is located between two of the previous situations.

STABILITY CHARTS – Bishop-Morgenstern's Charts

Normally, the user refers to an *average* r_u (along failure surface) given by:

$$\bar{r}_u = \frac{\sum_{i=1}^{n}\left(h_i\, r_{u,i}\right)}{\sum_{i=1}^{n}\left(h_i\right)}$$

where hi is the piezometric height of each of the slices in which the sliding mass can be subdivided.

Note: This method doesn't give the critical surface, but gives information about d_f.

Given a series of geometrical and geotechnical parameters, it does exist a r_u value indicated with $r_{u,e}$, through FOS for d_f = 1.0 is equal to FOS for d_f = 1.25, so that:

$$\begin{cases} F_1 = m_1 - n_1\, r_{u,e} \\ F_{1,25} = m_{1,25} - n_{1,25}\, r_{u,e} \end{cases} \rightarrow \quad r'_{u,e} = \frac{m_{1,25} - m_1}{n_{1,25} - n_1}$$

STABILITY CHARTS – Bishop-Morgenstern's Charts

Analogously, it exists a value of r_u which gives $F_{1.25} = F_{1.5}$, that is

$$r''_{u,e} = \frac{m_{1.5} - m_{1.25}}{n_{1.5} - n_{1.25}}$$

If the calculated average r_u is greater than $r'_{u,e}$, then $F_{1.25} < F_1$, so the condition with $d_f = 1.25$ is more critical.

The same reasoning can be made comparing user's situation with $d_f = 1.25$ condition and $d_f = 1.5$ condition for establish which is the most critical situation.

STABILITY CHARTS – Bishop-Morgenstern's Charts