Estimation of Bearing Capacity of Circular Footings on Sands Based on Cone Penetration Test

Junhwan Lee1 and Rodrigo Salgado2

Abstract: In the present paper, the estimation of limit unit bearing capacity \( q_{bl} \) of axially loaded circular footings on sands based on cone penetration test cone resistance \( q_c \) is examined. There are significant uncertainties in the calculation of bearing capacity using the bearing capacity equation. The selection of the value of the soil friction angle \( \phi \) and of the equation for \( N_q \) accounts for much of the overall uncertainty. The approach proposed in this study can reduce the uncertainties associated with the bearing capacity equation, as no estimation of \( \phi \) or selection of an equation for \( N_q \) is necessary. Instead, normalized limit unit bearing capacities \( q_{bl}/q_c \) are calculated from non-linear finite element and cone penetration resistance analyses for various soil and footing conditions. Effects of the relative density \( D_R \), the lateral earth pressure ratio \( K_o \) and the footing size are also addressed. It is observed that the normalized limit unit bearing capacity \( q_{bl}/q_c \) decreases as \( D_R \) increases and that this rate of decrease of \( q_{bl}/q_c \) varies with \( K_o \). Values of \( q_{bl}/q_c \) are presented both in equation form and in charts. The values of normalized allowable unit load \( q_{bl,all}/q_c \) at a settlement of 25 mm for various soil and stress states are also provided.

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Introduction

The bearing capacity problem of footings has been extensively studied for many decades. The bearing capacity equation expresses the unit load that would cause a footing to plunge into the ground as a function of the cohesive intercept \( c \), the surcharge \( q \) at the level of the footing base, and the unit weight \( \gamma \). A variety of factors, all functions of \( \phi \), appear in each term; in particular, factors \( N_c \), \( N_q \), and \( N_g \) appear multiplying \( c \), \( q \), and \( \gamma \), respectively. The pioneering work by Prandtl (1921) and Reissner (1924) established the values of the bearing capacity factors \( N_c \), \( N_q \), and \( N_g \) and most efforts since have focused on the evaluation of the bearing capacity factor \( N_c \) and correction factors associated with footing embedment, footing shape, and other complications (Terzaghi 1943; Brinch Hansen 1970; Vesic 1973; Kumbhojkar 1993; Michalowski 1997; Zhu et al. 2001).

For the limit unit bearing capacity \( q_{bl} \) of footings, important issues to be addressed include conservatism and uncertainty. The bearing capacity equation is based on the assumption of superposition of the \( c \), \( q \), and \( \gamma \) terms. This assumption leads to conservative values of \( q_{bl} \) (Griffiths 1982; Bolton and Lau 1993). The uncertainty results in large part from the uncertainties in the estimation of \( \phi \) and in the expression for \( N_g \). While the expressions for \( N_c \) and \( N_q \) are exact, there is no exact equation for \( N_g \). Additional uncertainty results from the expressions for the shape, depth, load inclination, and base and ground inclination factors, which are also approximate.

In addition to the uncertainty when estimating \( \phi \) from a given input, say relative density \( D_R \), the mobilized friction angle along the slip surface is not the same due to different dilatancy rates. This increases the uncertainty in the value of \( \phi \) to use in the bearing capacity equation, although the literature contains procedures for selection of values of the friction angle \( \phi \) that attempt to account for such effects (e.g., Perkins and Madson 2000).

According to Griffith (1982), the factor \( N_c \) represents the effect of the effective stress field below the footing base on \( q_{bl} \), and the initial value of the coefficient of lateral earth pressure \( K_o \), as a result, should affect the bearing capacity. However, using the two theorems of limit analysis (Drucker et al. 1952) to guide us, a different conclusion may be reached. From an upper bound point of view, the body forces are strictly vertical (gravity) forces; thus, in a cohesionless soil, gravity is the only source of resistance to collapse, and the values of the lateral stresses should not matter. From a lower bound point of view, the stress field that matters is one that is very close to collapse. So, again, \( K_o \) does not matter. So it seems that the only influence of \( K_o \) would be through the value of the resulting \( \phi \) value, which would be slightly less for a higher \( K_o \) value due to the increased confinement. Values of \( K_o \) for most sandy soils lie within the 0.4–0.5 range. The value of \( K_o=1.0 \) may be regarded as the upper limit for highly overconsolidated sands. An investigation of the effects of \( K_o \) on the bearing capacity is also desirable for improved designs.

The other uncertainty associated with \( N_g \) is in the evaluation of size effects. It has been shown and is intuitive that \( N_g \) varies with footing size, as the friction angle \( \phi \) decreases with the increasing confining stress associated with larger footings (Zhu et al. 2001).

In the present study, the evaluation of footing bearing capacity is examined with the aim to reduce these uncertainties. A method
Bearing Capacity Solutions for Footings

The bearing capacity equation for footings bearing on sand is

\[ q_{bl} = q_{n} d_{f} s_{f} b_{f} + \frac{1}{2} \gamma B N_{s} d_{f} i_{s} b_{f} \]  

(1)

where \( q_{bl} \) = limit unit bearing capacity; \( q \) = surcharge at footing base level; \( \gamma \) = unit weight of foundation soil; \( B \) = footing size; \( N_{s} \) and \( N_{q} \) = bearing capacity factors; and \( s, d, i, g, \) and \( b \) = shape, depth, load inclination, ground inclination, and base inclination factors, respectively. A wide range of \( N_{s} \) values exists from solutions based on limit equilibrium, limit analysis, and other numerical methods. The expression for \( N_{q} \), which is exact for weightless soil, is

\[ N_{q} = \frac{1 + \sin \phi}{1 - \sin \phi} (2) \]

The limit equilibrium method has been the most commonly used method to evaluate the bearing capacity of footings due to its simplicity (Terzaghi 1943; Vesic 1973; Kumbhojkar 1993; Zhu et al. 2001). According to typical slip patterns for limit equilibrium approaches shown in Fig. 1, a failure zone can be divided into active, radial shear, and passive zones. Zhu et al. (2001) performed limit equilibrium analyses using slip patterns with the angle \( \alpha \) of the active zone equal to \( \phi, 45^\circ + \phi/2 \), and an angle giving the minimum \( N_{q} \) value. The resulting equations for \( N_{q} \), corresponding to \( \alpha = \phi, 45^\circ + \phi/2 \), and the angle minimizing \( N_{q} \), are given by

\[ N_{q} = (2N_{q} + 1)(\tan \phi)^{1.35} \]  

(3)

\[ N_{q} = (2N_{q} + 1)(\tan 1.07\phi) \]  

(4)

\[ N_{q} = (2N_{q} + 1)(\tan \phi)^{1.45} \]  

(5)

The limit equilibrium approach does not produce mechanically rigorous solutions. Limit analysis solutions, on the other hand, are rigorous in the sense that the stress field of a lower bound solution is in equilibrium with imposed loads at the boundaries of the soil mass, while the velocity field of an upper bound solution is compatible with imposed velocities. The upper bound approach has frequently been used in the solution of footing bearing capacity problems (Chen 1975; Sloan and Yu 1996; Michalowski 1997; Soubra 1999; Zhu 2000). Michalowski (1997) presented equations for \( N_{q} \) developed for rough and smooth footings, respectively, based on upper bound solutions assuming multiblock failure mechanisms

\[ N_{q} = e^{0.66+5.11 \tan \phi} \tan \phi \]  

(6)

\[ N_{q} = e^{3.1 \tan \phi} \tan \phi \]  

(7)

The finite element and finite difference methods have been the most common numerical methods for analysis of the loading response and bearing capacity of footings (Griffiths 1982; Frydman and Burd 1997; Erickson and Drescher 2002). Griffiths (1982) used the finite element analysis for the determination of the bearing capacity factors and assessment of the Terzaghi bearing capacity equation. Values of the bearing capacity factors evaluated by Griffiths (1982) are in good agreement with those obtained by Prandtl (1921) and Terzaghi (1943) using limit equilibrium methods. Frydman and Burd (1997) and Erickson and Drescher (2002) performed finite difference analyses. Results of the analyses showed that the bearing capacity factors increase as the soil dilatancy angle \( \psi \) increases. This also indicates that the dilatancy angle \( \psi \) should be considered in the estimation of bearing capacity when high accuracy is needed.

Numerical Simulation of Footing Load Tests

A modern design concept in geotechnical engineering is to treat serviceability and ultimate limit states within the same framework. In this integrated framework, the stability and functionality of foundations are not independent, and both strength and stiffness are properly taken into account. In this study, therefore, the numerical simulation of footing load response and bearing capacity is based on a stress–strain model that represents the pre- and postfailure behavior of soils realistically, considering the nonlinearity of soil stiffness and strength.

Nonlinear Stiffness and Strength Model

Based on the observed degradation of elastic modulus in sands, Lee and Salgado (2000) suggested the following modulus degradation relationship for a general stress state:

\[ \frac{G}{G_{0}} = \left[ 1 - f \left( \frac{\sqrt{J_{2}} - \sqrt{J_{20}}}{\sqrt{J_{2\max}} - \sqrt{J_{20}}} \right)^{2} \right] \left[ \frac{I_{1}}{I_{10}} \right]^{n_{g}} \]  

(8)

where \( G \) and \( G_{0} \) = initial and secant shear modulus; \( J_{2}, J_{20}, \) and \( J_{2\max} \) = current, initial, and maximum second invariants of the deviatoric stress tensor; \( I_{1} \) and \( I_{10} \) = first invariants of the stress tensor at the current and initial states; \( f \) and \( g \) = material parameters that vary as a function of \( D_{k} \); and \( n_{g} \) = material parameters representing effect of confining stress. There are several ways to obtain values of \( G_{0} \) at small strain including empirical equations, laboratory tests and in situ tests. In this study, the following empirical equation, based on the work of Hardin and Black (1966), was used to estimate \( G_{0} \) for various soil states

\[ G_{0} = C_{e} \frac{e_{c} - e_{0}}{1 + e_{0}} p_{A} \left( \frac{\alpha_{m}}{a_{m}} \right)^{n_{g}} \]  

(9)

where \( C_{e}, n_{g}, \) and \( e_{c} \) = intrinsic material variables; \( e_{0} \) = initial void ratio; \( p_{A} \) = reference pressure = 100 kPa; and \( \alpha_{m} \) = initial mean effective stress in the same unit as \( p_{A} \).

In order to describe failure and post-failure soil responses, the Drucker–Prager failure criterion was adopted. It has been well
known that the peak friction angle \( \phi_p \) defined in terms of the critical state friction angle \( \phi_c \) and the peak dilatancy angle \( \psi_p \), is a stress- and density-dependent variable (Bolton 1986). While \( \phi_c \) is an intrinsic soil variable, independent of stress state, history, and density, \( \psi_p \) varies with both relative density and confinement. As a result, the envelope of the peak failure surface is nonlinear.

In this study, the following relationship proposed by Bolton (1986) was used to estimate the peak friction angle \( \phi_p \) in sand:

\[
\phi_p = \phi_c + 0.8 \psi_p
\]

(10)

where \( \phi_p \) = peak friction angle; \( \phi_c \) = critical state friction angle; and \( \psi_p \) = peak dilatancy angle = 6.25 \( I_R \) and 3.75 \( I_R \) for plane-strain and triaxial conditions, respectively. The dilatancy index \( I_R \) is given by

\[
I_R = I_D \left[ Q - \ln \left( \frac{100p_s}{p_A} \right) \right] - R
\]

(11)

where \( I_D \) = relative density (as a number between 0 and 1); \( p_A \) = reference pressure = 100 kPa; \( p_s' \) = mean effective stress at peak strength in the same units as \( p_A \); and \( Q \) and \( R \) = intrinsic soil variables. Eqs. (10) and (11) were used to define the nonlinear Drucker–Prager failure surface in the numerical simulation of footing load responses.

**Numerical Modeling of Footing Load Tests**

The finite element (FE) method was used to model axially loaded circular footings on sands. The commercial FE program ABAQUS was used to model the footing load tests, with a subroutine specifically written for the nonlinear stiffness and strength model described previously. Eight-noded axisymmetric elements were used in the finite element meshes to model both soils and the footings. For realistic numerical simulation of footing load response, the variation of \( \phi_p \) of the foundation soil with confining stress must be considered. Even for a homogeneous deposit of constant \( D_R \), the peak friction angle \( \phi_w \) would vary with depth due to different confining stresses. At the same depth, \( \phi_p \) would also vary with footing size as a result of the different stress states generated by the footing. The friction angle \( \phi_p \) in our finite element analyses is therefore a variable, reflecting depth, stress state and footing size, rather than a constant value.

The degree of roughness between the footing base and soil is another factor to be addressed in the analysis. It is reasonable to expect that values of \( N_c \) increase as the footing roughness increases. While many authors proposed values of \( N_c \) for rough and smooth conditions separately, the actual footing condition would be neither perfectly rough nor perfectly smooth. Accordingly, in order to realistically model footing roughness, interface elements were used between the footing base and soil with a Coulomb friction coefficient of 0.7, corresponding to an interface friction angle equal to about 35°. This interface allows sliding between the footing and the soil whenever the interface shear strength is exceeded.

**Measured and Predicted Footing Loading Responses**

In order to validate the finite element analysis, field test results on axially loaded footings in sands were selected from the literature for comparison with finite element predictions. Footing load tests were performed at Texas A&M Univ. for the Settlement ‘94 ASCE Special Conference (Briaud and Jeanjean 1994). Square footings with four different sizes were tested: 1 × 1, 1.5 × 1.5, 2.5 × 2.5, and 3 × 3 m footings. The test site consists predominantly of sands down to a depth of 11 m. Beneath this sand layer, there is a very stiff clay deposit extending down to a depth of approximately 33 m. The water table was found at a depth equal to around 4.9 m.

As the test footings were square, both footings and soils were modeled with 20-noded three-dimensional (3D) solid elements instead of axisymmetric elements. Interface elements were used between footings and the surrounding soil. As the test footings were embedded 0.76 m into the soil, the same embedment was modeled in the finite element analyses for all the footings. This embedment corresponds to a soil overburden pressure equal to approximately 11.6 kPa. The initial stress states for the analyses were set as geostatic, based on unit weights estimated from the information obtained from the site characterization. All the soil properties used in the finite element analysis were those obtained from the site characterization and laboratory tests performed before the conference and provided in Briaud and Jeanjean (1994). Fig. 2 shows the measured and predicted footing load responses, where \( q_f \) represents the footing unit load. For comparison, load–settlement results using Schmertmann’s method (Schmertmann et al. 1978) are also plotted. As can be seen in the figure, predictions using the nonlinear FE analyses show reasonably good agreement with measured results.

**Footing Bearing Capacity for Various Soil States**

**Development of Footing Load–Settlement Curves**

In order to obtain footing bearing capacities for a variety of soil states and footing sizes, a series of FE analyses were performed. Footings of different diameters (\( B = 1, 2, \) and 3 m) with stiffness much greater than that of the soil were modeled as resting on the soil surface with no surcharge. The soil was assumed to be Ottawa sand, whose properties have been widely studied. Four different relative densities (\( D_R = 30, 50, 70, \) and 90%) and three lateral earth pressure ratios (\( K_0 = 0.45, 0.70, \) and 1.00) were used in the analyses. The construction of the finite element models was done in a manner similar to that described in the previous section. The lateral and bottom boundaries of the finite element meshes were located at 12 m horizontally and 15 m vertically from the center of the footing base, respectively. Based on analyses with meshes of various sizes, it was found that the mesh size used in this study, extending laterally to more than four times the footing diameter and vertically to more than five times the footing diameter, is large enough to eliminate boundary effects. Interface elements were also used between the footing base and the soil, with a Coulomb friction coefficient of 0.7, corresponding to a friction angle equal to about 35°. Fig. 3 shows a typical finite element mesh used in this study.

Fig. 4 shows load-settlement curves from the finite element footing load analyses for different footing sizes (\( B = 1, 2, \) and 3 m) and relative densities (\( D_R = 30, 50, 70, \) and 90%) under \( K_0 = 0.45 \). All the load–settlement curves were developed up to a settlement level equal to 20% of the footing diameter (i.e., \( s/B = 0.2 \)), corresponding to 20, 40, and 60 cm for the 1, 2, and 3 m footings, respectively. The footing unit load \( q_f \) at \( s/B = 0.2 \) is taken here, for simplicity, as the limit unit bearing capacity \( q_{ud} \) of the footing. Note that this is not strictly the limit unit bearing capacity \( q_{ud} \) of the footing, particularly in dense sands, for which a considerable increase in bearing capacity is still possible after 20% settlement; however, this larger bearing capacity is ulti-

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mately irrelevant for practical purposes, given the large settlements associated with it. Footing load responses for soils of $K_0 = 0.70$ and $1.00$ were also developed using the same footing diameters and relative densities as used for $K_0 = 0.45$. Although not shown here, the shapes of the load–settlement curves for $K_0 = 0.70$ and $1.00$ are similar to those for $K_0 = 0.45$.

Fig. 5 shows values of limit unit bearing capacity $q_{bl}$ obtained from the finite element analysis for different footing diameters and soil states. It is observed that the effect of $D_R$ on the limit unit bearing capacity $q_{bl}$ is significant, while that of $K_0$ is relatively small, which is as expected based on considerations made earlier in the paper. This result provides support for neglecting $K_0$ effects, as most bearing capacity solutions do.

**Bearing Capacities from Finite Element Method and Bearing Capacity Factors**

The limit unit bearing capacity values obtained from the finite element analysis in this study were compared with those calculated using the bearing capacity factor $N_g$. Selected $N_g$ values for the comparison include those of Brinch Hansen (1970), Vesic (1973), Bolton and Lau (1993), and Michalowski (1997). The $N_g$ values of Michalowski (1997) are based on upper bound limit analysis, while those by Brinch Hansen (1970), Vesic (1973), and Bolton and Lau (1993) are based on limit equilibrium solutions.

Table 1 shows values of $N_g$ calculated following Brinch Hansen (1970), Vesic (1973), Bolton and Lau (1993), and Michalowski (1997) as a function of the friction angle $f$. The $N_g$ values by Bolton and Lau (1993) differ significantly depending on whether smooth or rough conditions are assumed. As discussed before, the actual footing base condition would be neither completely smooth nor rough. Hence, the average of the $N_g$ values calculated by Bolton and Lau (1993) for smooth and rough conditions is also given in the table.

Fig. 6 shows limit bearing capacities $q_{bl}$ of footings obtained from the finite element analysis and the various $N_g$ values for $K_0 = 0.45$. The friction angles $\phi$ for the determination of $N_g$ values in Fig. 6 were obtained from Eqs. (10) and (11) for the soil states considered. This requires the values of relative density $D_R$, which are known, and peak mean effective stress $p'_e$. Several different expressions have been suggested for the estimation of $p'_e$ for use in connection with the bearing capacity problem (Meyerhof 1950; De Beer 1965; Perkins and Madson 2000). In this study, the following relationship by Perkins and Madson (2000) is used:

![Image of finite element mesh for axially loaded circular footing](image)
Fig. 4. Load–settlement curves with $K_0=0.45$ for: (a) 1 m footing, (b) 2 m footing, and (c) 3 m footing

Fig. 5. Limit unit bearing capacity for different values of $D_R$, $K_0$, and footing diameter $B$

\begin{equation}
\frac{p'_u}{q_{bl}} = \frac{1}{6} \left( 0.52 - 0.04 \frac{L}{B} \right)
\end{equation}

where $p'_u$=peak mean effective stress; $q_{bl}$=limit unit bearing capacity; and $L$ and $B$=length and width of footing, respectively. The use of Eq. (12) is primarily due to simplicity while producing general agreement with previously published work and flexibility for 3D stress conditions, as explained by Perkins and Madson (2000). Values of $q_{bl}$ in Eq. (12) for different methods were obtained iteratively with $N_v$ values corresponding to each method shown in Fig. 6. As shown in the figure, depending on the values of $N_v$, large variations in $q_{bl}$ are observed. It is seen that the values of $q_{bl}$ calculated using $N_v$ by Bolton and Lau (1993) for the rough case are significantly greater than those from the finite element analysis. Use of $N_v$ values from Brinch Hansen (1970), Vesic (1973), Michalowski (1997), and Bolton and Lau (1993) for the smooth case, on the other hand, produces smaller $q_{bl}$ values than the finite element results. Use of $N_v$ values obtained by averaging the smooth and rough base values of $N_v$ from Bolton and Lau (1993) results in reasonable agreement with the finite element results. Similar results were observed for $K_0=0.70$ and 1.00.

**Cone Penetration Test-Based Footing Bearing Capacity**

**Normalized Limit Unit Bearing Capacity $q_{bl}/q_c$**

In order to obtain footing bearing capacity based on CPT cone resistance $q_c$, the load–settlement curves obtained from the finite element analysis were normalized in terms of the normalized footing unit load $q/L$ and the relative settlement $s/B$. Values of $q_c$ used to normalize the load–settlement curves were obtained from CONPOINT, which is a cone resistance analysis program (Salgado and Randolph 2001; Salgado 2003) that can be used to calculate the cone resistance $q_c$. The cone resistance $q_c$ from CONPOINT is determined using cavity expansion theory. For un-cemented granular soils, it is generally possible to write

\begin{equation}
q_c = q_0(D_R, \sigma'_h, \sigma'_v)
\end{equation}

where $q_0$=function containing intrinsic variables; $D_R$=relative density of sand before penetration; and $\sigma'_h$ and $\sigma'_v$=initial vertical and lateral effective stresses. The details of the theoretical development, evaluation, and validation of the function represented by Eq. (13) are available in Salgado et al. (1997), Salgado and Randolph (2001), and Salgado (2003). The values of the intrinsic and state variables of the soil used in the CONPOINT analysis were the same as used in the finite element analysis.

The critical slip surface for footings is observed to extend to a depth, measured from the footing base, approximately equal to the footing diameter $B$. Accordingly, cone resistance values used to normalize the load–settlement curves were average cone resistances $q_c,_{avg}$ from the footing base to a depth $B$ below the footing base. The load–settlement curves for $K_0=0.70$ and 1.00 were normalized in the same way.

Fig. 7 shows normalized footing unit load $q/L$ versus relative settlement $s/B$ for $K_0=0.45$ and different values of $D_R$ and $B$. Normalized load–settlement curves for $K_0=0.70$ and 1.00 were also developed. From the normalized load–settlement curves, values of the normalized limit unit bearing capacity $q_{bl}/q_c$ were determined at $s/B=0.2$. 

\[
N_v = \frac{1}{6} \left( 0.52 - 0.04 \frac{L}{B} \right)
\]
Effects of $K_0$ and $D_R$ on Normalized Limit Unit Bearing Capacity $q_{BL}/q_c$

As discussed earlier, the influence of $D_R$ on the limit unit bearing capacity $q_{BL}$ is significant while that of $K_0$ is small to negligible. For the normalized limit unit bearing capacity $q_{BL}/q_c$, however, effects of both $D_R$ and $K_0$ are found to be significant. Fig. 8 shows values of normalized limit unit bearing capacity $q_{BL}/q_c$ for a 1 m footing as a function of $D_R$ and $K_0$. From the figure, it is observed that values of $q_{BL}/q_c$ decrease as $D_R$ increases and that the rate of decrease depends on the value of $K_0$. At $D_R=30\%$, values of $q_{BL}/q_c$ are equal to 0.36 and 0.25 for $K_0=0.45$ and 1.00, while those at $D_R=90\%$ are 0.22 and 0.20, respectively. This indicates that the effect of $K_0$ on $q_{BL}/q_c$ becomes smaller as $D_R$ increases.

The dependence of $q_{BL}/q_c$ on $K_0$ is mainly due to the dependence of the cone resistance $q_c$ on the horizontal effective stress $s'_h$ and thus, on $K_0$. It is known that the horizontal effective stress $s'_h$ rather than the vertical effective stress $s'_v$ is a key variable along with the $D_R$ in the determination of $q_c$ (Salgado et al. 1997). As a result, the normalized limit unit bearing capacity $q_{BL}/q_c$ shows a strong dependence on $K_0$, while the limit unit bearing capacity $q_{BL}$ is insensitive to $K_0$.

### Table 1. Values of $N_d$ Proposed by Different Authors

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**Fig. 6.** Comparison of $q_{BL}$ from finite element analyses and bearing capacity equation

**Fig. 7.** Normalized load–settlement curves in terms of $q_{BL}/q_c$ and $s/B$ for: (a) 1 m footing, (b) 2 m footing, and (c) 3 m footing
The issue of scale effects in the bearing capacity of footings, particularly with respect to the factor $N_v$, must also be addressed. Several authors investigated scale effects in bearing capacity (Perkins and Madson 2000; Zhu et al. 2001). It is observed that the limit unit bearing capacity $q_{bl}$ does not increase linearly with increasing footing size $B$, as the bearing capacity equation would appear to suggest. This scale effect is primarily due to the magnitude of the mobilized soil friction angle. As the footing size increases, producing larger confining stresses in the soil, the mobilized friction angle decreases, resulting in nonlinear increases of $q_{bl}$ with $B$. Similar trends are observed in the normalized limit unit bearing capacity. Fig. 9 shows values of $q_{bl}/q_c$ as a function of the footing diameter $B$. As shown in the figure, the normalized limit unit bearing capacity $q_{bl}/q_c$ increases nonlinearly with increasing footing diameter $B$. However, it should be noticed that the scale effect for $q_{bl}/q_c$ is not as significant as for $q_{bl}$, since the normalization based on $q_{c,avg}$ within an influence depth equal to $B$ already reflects, in part, the effects of the size of the footing.

### Allowable Footing Unit Load at Common Design Settlements

In design, settlement considerations often control. The tolerable settlement $s_{tol}$ for shallow foundations is commonly assumed as 25 mm (or 1 in) with some variation depending on the importance of the structure (Terzaghi and Peck 1967; Becker 1996). Fig. 10 shows values of allowable footing unit load $q_{b,all}$ at $s_{tol}=25$ mm as a function of soil density [as reflected by $D_R$ and standard penetration test (SPT) blow counts $N$] and footing size. The solid lines are from our finite element analysis; the dashed lines are from field test results by Terzaghi and Peck (1967).

As can be seen in the figure, $q_{b,all}$ becomes more sensitive to footing size as $D_R$ increases. At $D_R=90\%$, values of $q_{b,all}$ at $s_{tol}=25$ mm are 0.54, 0.47, and 0.44 MPa for the 1, 2, and 3 m footings under $K_0=0.45$, respectively, while at $D_R=30\%$ the $q_{b,all}$ is insensitive to footing size. It is also shown that the effect of $K_0$ on the value of $q_{b,all}$ is greater at higher $D_R$ values. From the results of Fig. 10, it is seen that results obtained in this study match well the results from the field tests of Terzaghi and Peck (1967), both of which show increasing dependence of $q_{b,all}$ on footing size as the $D_R$ increases.

Normalized allowable unit load $q_{b,all}/q_c$ at 25 mm settlement was also obtained. Fig. 11 shows values of $q_{b,all}/q_c$ at the 25 mm settlement as a function of $B$, $D_R$, and $K_0$. As shown in the figure, values of $q_{b,all}/q_c$ decrease with increasing footing diameter. The size effect observed in Fig. 11 is consistent with that for the allowable footing unit load $q_{b,all}$ shown in Fig. 10. The influence of $K_0$ on values of $q_{b,all}/q_c$ was, however, different, depending on the values of $D_R$. At $D_R=90\%$, values of $q_{b,all}/q_c$, shown as thick lines in Figs. 11(a–c), are in nearly the same range of 0.05–0.08 for all the values of $K_0$ considered. At $D_R=30\%$, on the other hand, values of $q_{b,all}/q_c$ shown as thin lines in Figs. 11(a–c) decrease as $K_0$ increases. Note that if $s_{tol}=25$ mm, serviceability always controls for $B=1$–3 m, as $q_{b,all}/q_c$ is always less than $q_{b,all}$. 

### Design Application

Estimation of the normalized limit unit bearing capacity $q_{bl}/q_c$ for circular footings in sands can be made directly by applying the results of Figs. 8 and 9 for a given soil state and footing size. The CPT-based bearing capacity obtained in this study can be expressed as follows:

$$q_{bl} = \beta \cdot q_{c,avg}$$  \hspace{1cm} (14)

where $q_{bl}$=limit unit bearing capacity; $\beta$=correlation factor; and $q_{c,avg}$=average cone resistance from the footing base to a depth measured from the footing base equal to the footing diameter $B$.

Based on the results of Figs. 8 and 9, values of $\beta$ are determined...
and summarized in Table 2. As shown in Fig. 9, the effect of footing size appears to be more significant as the $D_R$ increases. Although some increase of $b$ with increasing $B$ is observed for looser soils (i.e., $D_R \approx 50\%$), this increase may be ignored for practical purposes. Consequently, the values of $b$ in Table 2 reflect size effects only for denser sands (i.e., $D_R \approx 70\%$).

Tand et al. (1995) and Eslamizad and Robertson (1996) also proposed equations similar to Eq. (14) (Lunne et al. 1997). Most existing equations, however, do not include the effects of soil state as expressed by the relative density $D_R$ and the lateral earth pressure ratio $K_0$ on the values of $b$. For instance, values of $b$ according to Tand et al. (1995) vary from 0.14 to 0.2 in terms of footing shape and depth. Briaud and Jeanjean (1994) proposed a value of $b$ equal to 0.25 based on the Texas A&M footing load test results previously described. It is noticeable that the value of $b$ equal to 0.25 proposed by Briaud and Jeanjean (1994) corresponds to a value of $b$ from Table 2 corresponding to a lightly to moderately overconsolidated $K_0 \approx 0.45 – 0.7$, medium dense ($D_R \approx 70\%$) sand, which correspond to a soil state that is indeed similar to the actual soil state at the Texas A&M site, where their load tests were performed.

The results given in Table 2 are for circular footings on sands without a surcharge. Application of the results to square footings is possible with minimal error so long as an equivalent area is considered. For footing shapes other than circular or square, introduction of shape factors would be required.

**Examples**

In order to illustrate the use of the proposed method, a second site was selected from the literature (Lee et al. 2003) and used for the estimation of footing bearing capacity. The soil at the site is a predominantly gravelly sand down to a depth of around 13–14 m. Results from the site investigation indicate that the first 3 m of the gravelly sand deposit are in a loose state ($D_R < 30\%$, $\gamma < 16.5 \text{kN/m}^3$, and $\phi_c < 33\deg$), while the rest of the deposit down to a depth of 13–14 m is in dense to very dense states ($D_R < 80\%$, $\gamma < 18.0 \text{kN/m}^3$, and $\phi_c < 33\deg$). Soils near the surface are likely overconsolidated due to removal of a pavement and fill materials that existed there previously before the cone penetration tests and the rest of the ground characterization were conducted. Three circular footings, with diameters equal to 1, 2, and 3 m, are considered in this example.

The method proposed in this study can be used for estimating the bearing capacity of these footings by following these steps:

1. estimation of representative values of relative density $D_R$ and lateral earth pressure ratio $K_0$ within the influence depth below the footing base (equal to the footing diameter $B$);

<table>
<thead>
<tr>
<th>$D_R$</th>
<th>$K_0=0.45$</th>
<th>$K_0=0.70$</th>
<th>$K_0=1.00$</th>
</tr>
</thead>
<tbody>
<tr>
<td>30%</td>
<td>0.36</td>
<td>0.29</td>
<td>0.25</td>
</tr>
<tr>
<td>50%</td>
<td>0.30</td>
<td>0.27</td>
<td>0.24</td>
</tr>
<tr>
<td>70%</td>
<td>0.26</td>
<td>0.25</td>
<td>0.23</td>
</tr>
<tr>
<td>$B&gt;B_r^a$</td>
<td>0.26 (0.90+0.10B/$B_r$)</td>
<td>0.25 (0.92+0.08B/$B_r$)</td>
<td>0.23 (0.94+0.07B/$B_r$)</td>
</tr>
<tr>
<td>$B&gt;B_r^a$</td>
<td>0.22</td>
<td>0.21</td>
<td>0.19</td>
</tr>
<tr>
<td>$B=B_r^a$</td>
<td>0.22 (0.85+0.15B/$B_r$)</td>
<td>0.21 (0.85+0.15B/$B_r$)</td>
<td>0.19 (0.85+0.15B/$B_r$)</td>
</tr>
</tbody>
</table>

$^a$Reference footing diameter=1 m = 3.28 ft.

![Fig. 11. Normalized allowable unit load $q_{b,all}/q_c$ at 25 mm settlement versus footing diameter $B$ for: (a) $K_0=0.45$, (b) $K_0=0.70$, and (c) $K_0=1.00$](image-url)
2. determination of \(\beta\) from Table 2 for the given values of \(D_R\) and \(K_0\);
3. estimation of average cone resistance \(q_c,\text{avg}\) within the influence depth (equal to \(B\)); and
4. determination of limit unit bearing capacity \(q_{bl}\) using Eq. (14).

Fig. 12 shows the \(q_c\) profile obtained from cone penetration testing performed at the site. From Table 2, the value of \(\beta\) was estimated as 0.31 (corresponding to \(D_R = 30\)–40% and \(K_0\) value equal to 0.6–0.7, with assumed overconsolidation ratio equal to around 2–3 to account for the surface overconsolidation condition). No size correction is necessary, as the relative density is lower than 50%. As shown in Fig. 12, values of \(q_c,\text{avg}\) within the influence depths are 2.52, 2.53, and 3.53 MPa for 1, 2, and 3 m footings, respectively. From the values of \(\beta\) and \(q_c,\text{avg}\), the values of limit unit bearing capacity \(q_{bl}\) were estimated as 0.78, 0.79, and 1.09 MPa for the 1, 2, and 3 m footings, respectively.

For comparison purposes, \(q_{bl}\) was also estimated using the bearing capacity factor \(N_p\). The values of \(N_p\) according to the equation put forth by Vesic (1973) were 79.9, 71.3, and 66.7 for the 1, 2, and 3 m footings, respectively. These values were obtained based on peak friction angles \(\phi_c\) equal to 38.2, 37.5, and 37.1° estimated from Eqs. (10)–(12) with \(D_R = 30\)–40%, \(\gamma = 16.5\) kN/m³, and \(\phi_r = 33°\). Calculated values of \(q_{bl}\) are then 0.40, 0.71, and 0.99 MPa for the 1, 2, and 3 m footings, respectively. Compared with the values obtained using the proposed method, these are conservative, particularly for the 1 m footing. If a factor of safety (FS) equal to 3.0 is introduced, values of the allowable unit bearing capacity \(q_{bl,\text{all}}\) are 0.13, 0.24, and 0.33 MPa for the 1, 2, and 3 m footings, respectively.

From Fig. 10, it can be seen that the value of \(q_{bl,\text{all}}\) for the 1 m footing is close to the value of the allowable bearing unit load for a 25 mm settlement, \(q_{bl,\text{all}} \approx 0.12\) MPa, corresponding to \(B = 1\) m and \(D_R = 30\%). For the larger footings (i.e., \(B = 2\) and \(3\) m), on the other hand, the values of \(q_{bl,\text{all}}\) equal to 0.24 and 0.33 MPa are greater than values of \(q_{bl}\) shown in Fig. 10 by a large margin. This indicates that, as the footing size and relative density decrease, the limit bearing capacity limit state controls design, while the settlement-based allowable unit load \(q_{bl,\text{all}}\) controls for larger footings in denser soils. This result is also shown in Fig. 13, in which the ratio of \(q_{bl,\text{all}}/q_{bl,\text{all}}\) with footing size and relative density.

**Summary and Conclusions**

In the present study, estimation of the bearing capacity of circular footings on sands based on CPT cone resistance \(q_c\) was investigated. A conventional check of the bearing capacity limit state using the bearing capacity equation requires calculation of \(N_p\) and thus an estimate of \(\phi\). Significant uncertainties exist in the values of \(N_p\) and the soil friction angle \(\phi\). The CPT-based bearing capacity analysis of this study does not require estimates of \(\phi\) or \(N_p\); bearing capacity is determined directly from the CPT cone resistance.

In order to simulate the loading response of axially loaded circular footings in sands, the finite element method was used with a subroutine written for the non-linear elastic-plastic stress-strain model of Lee and Salgado (2000). Various soil states, including four different relative densities (\(D_R = 30, 50, 70, \) and 90%) and three different lateral stress ratios (\(K_0 = 0.45, 0.70, \) and 1.00), were considered. The footings were modeled as elastic with three different diameters (\(B = 1, 2, \) and 3 m). As the footing base is neither perfectly smooth nor perfectly rough, interface elements between the footing and soil were used. Based on the finite element results, load–settlement curves were developed and used to determine the limit unit bearing capacity \(q_{bl}\) for various soil and footing conditions.

In order to obtain the normalized limit unit bearing capacity \(q_{bl}/q_c\), the load–settlement curves obtained from the finite element analysis were normalized with respect to cone resistance \(q_c\). The cone resistance \(q_c\) used for the normalization was an average value for the influence zone extending from the footing base to a depth equal to the footing diameter \(B\) below the footing base. Values of \(q_{bl}/q_c\) were found to be in a range of 0.20–0.36, depending on the values of \(D_R, K_0,\) and \(B\). It was found that both the relative density \(D_R\) and the lateral earth pressure ratio \(K_0\) are
important factors affecting the value of the normalized limit unit bearing capacity \(q_{bl}/q_c\). The normalized limit unit bearing capacity \(q_{bl}/q_c\) decreases as \(D_R\) increases, with the rate of decrease depending on the \(K_0\) values. The effect of \(K_0\) was greater for lower \(D_R\) values.

There is a size effect on the values of \(q_{bl}/q_c\). The size effect increases as the footing diameter \(B\) increases. For \(D_R\) lower than about 50%, the size effect on \(q_{bl}/q_c\) may be ignored for practical purposes. Design values of the normalized limit unit bearing capacity were presented in equation, chart, and table formats.

The normalized allowable unit load \(q_{bl,all}/q_c\) at 25 mm settlement was also studied. Results from the present analysis are in close agreement with field measurements. The normalized allowable unit loads at other settlement levels can also be determined based on the normalized load–settlement curves in terms of \(q_{bl}/q_c\) versus \(s/B\).

### Notation

The following symbols are used in this paper:

- \(B\) = footing width or diameter;
- \(b_c, b_q, b_i\) = base inclination factors;
- \(C_f\) = small-strain shear modulus number;
- \(c\) = cohesive intercept;
- \(D_R\) = relative density (%);
- \(d_c, d_q, d_i\) = depth factors;
- \(e_f\) = small-strain shear modulus void ratio number;
- \(e_{0i}\) = initial void ratio;
- \(f\) = material parameter in nonlinear stiffness model;
- \(G\) = secant shear modulus;
- \(G_0\) = initial shear modulus;
- \(g\) = material parameter in nonlinear stiffness model;
- \(g_c, g_q, g_i\) = ground inclination factors;
- \(I_p\) = relative density as number between 0 and 1;
- \(I_d\) = dilatancy index;
- \(I_1\) = first invariant of stress tensor at current state;
- \(I_{10}\) = first invariant of stress tensor at initial state;
- \(i_c, i_q, i_{10}\) = load inclination factors;
- \(J_2\) = second invariant of deviatoric stress tensor at current state;
- \(J_{2\text{max}}\) = second invariant of deviatoric stress tensor at failure;
- \(J_{20}\) = second invariant of deviatoric stress tensor at initial state;
- \(K_0\) = coefficient of lateral earth pressure at rest;
- \(L\) = footing length;
- \(N_c, N_q, N_i\) = bearing capacity factors;
- \(n_f\) = small-strain shear modulus exponent;
- \(p_{at}\) = atmospheric pressure;
- \(p_{ef}\) = mean effective stress at peak strength;
- \(Q\) = intrinsic soil variable in correlation for peak friction angle;
- \(q\) = surcharge;
- \(q_{bl}\) = footing unit load;
- \(q_{bl,all}\) = allowable footing unit load;
- \(q_{bl,all,B}\) = allowable unit bearing capacity of footing;
- \(q_{bl}\) = limit unit bearing capacity of footing;
- \(q_c\) = Cone penetration test cone resistance;
- \(q_{c,avg}\) = average Cone penetration test cone resistance within influence depth;
- \(R\) = intrinsic soil variable in correlation for peak friction angle;
- \(s\) = footing settlement;
- \(s_{at}, s_{q}, s_{q}\) = shape factors;
- \(s_{tol}\) = tolerable footing settlement;
- \(s/B\) = relative settlement (settlement divided by footing size);
- \(\beta\) = correlation factor between cone resistance and footing bearing capacity;
- \(\gamma\) = soil unit weight;
- \(\sigma_{ef}\) = horizontal effective stress;
- \(\sigma_{me}\) = mean effective stress;
- \(\sigma_{ve}\) = vertical effective stress;
- \(\phi\) = friction angle;
- \(\phi_c\) = friction angle at critical state;
- \(\phi_p\) = peak friction angle; and
- \(\psi_p\) = peak dilatancy angle.

### References


on bearing capacity using limit analysis.” *Soils Found.*, 37(4), 57–64.


